Formation of a quadratic equation:

If α and β are the roots of the required quadratic equation.

Let $x = \alpha$ and $x = \beta$

i.e., $x - \alpha = 0$, $x - \beta = 0$

and $(x - \alpha)(x - \beta) = 0$

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

which is the required quadratic equation in standard form.

Find a quadratic equation from given roots and establish the formula

$x^2 - (\text{sum of the roots})x + \text{product of the roots} = 0$

Let α, β be the roots of the given equation

$ax^2 + bx + c = 0 \quad , \quad (a \neq 0) \quad ....(i)$

Then $\alpha + \beta = -\frac{b}{a}$ *and* $\alpha\beta = \frac{c}{a}$

Rewrite eq.(i) as $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

or $x^2 - \left(-\frac{b}{a}\right)x + \frac{c}{a} = 0$

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

or $x^2 - (\text{sum of the roots})x + \text{product of the roots} = 0$, that is

$x^2 - Sx + P = 0$ where $S = \alpha + \beta$ and $P = \alpha\beta$

Exercise 2.5

1. Write the quadratic equations having following roots.

(a) 1,5

Solution:

S = Sum of roots = 1 + 5 = 6

P = Product of roots = (1) (5) = 5

Thus, the required quadratic equation is

$x^2 - Sx + P = 0$

$x^2 - 6x + 5 = 0$

(b) 4,9

Solution:

Since 4 and 9 are the roots of the required quadratic equation, therefore,

S = Sum of roots = 4 + 9 = 13

P = Product of roots = (4) (9) = 36

Thus, the required quadratic equation is

$x^2 - Sx + P = 0$

$x^2 - 13x + 36 = 0$

(c) -2,3

Solution:

Since -2 and 3 are the roots of the required quadratic equation, therefore,

S = Sum of roots = -2 + 3 = 1

P = Product of roots = (-2) (3) = -6

Thus, the required quadratic equation is

$x^2 - Sx + P = 0$

$x^2 - (1)x + (-6) = 0$

$x^2 - x - 6 = 0$

Solution:

Since 0 and – 3 are the roots of the required quadratic equation, therefore,

S = Sum of roots = 0 + (– 3) = – 3

P = Product of roots = (0) (– 3) = 0

Thus, the required quadratic equation is

$x^2 - Sx + P = 0$

$x^2 - (-3)x + 0 = 0$

$x^2 + 3x = 0$

(e) 2, – 6

Solution:

Since 2 and – 6 are the roots of the required quadratic equation, therefore,

S = Sum of roots = 2 + (– 6) = – 4

P = Product of roots = (2) (– 6) = – 12

Thus, the required quadratic equation is

$x^2 - Sx + P = 0$

$x^2 - (-4)x + (-12) = 0$

$x^2 + 4x - 12 = 0$

(f) – 1, – 7

Solution:

Since –1 and – 7 are the roots of the required quadratic equation, therefore,

S = Sum of roots = (–1) + (– 7) = – 1– 7 = – 8

P = Product of roots = (–1) (– 7) = 7

Thus, the required quadratic equation is

$x^2 - (-8)x + 7 = 0$

$x^2 + 8x + 7 = 0$

(g) 1+ i, 1 – i

Solution:

Since 1+ i and 1 – i are the roots of the required quadratic equation, therefore,

S = Sum of roots = (1+ i) + (1 – i) = 2

P = Product of roots = (1+ i) (1 – i) = $1 - i^2 = 1 - (-1) = 2$

Thus, the required quadratic equation is

$x^2 - Sx + P = 0$

$x^2 - 2x + 2 = 0$

(h) 3+ $\sqrt{2}$, 3 – $\sqrt{2}$

Solution:

Since 3+ $\sqrt{2}$ and 3 – $\sqrt{2}$ are the roots of the required quadratic equation, therefore,

S = Sum of roots = $3 + \sqrt{2} + 3 - \sqrt{2} = 6$

P = Product of roots = $(3 + \sqrt{2})(3 - \sqrt{2}) = (3)^2 - (\sqrt{2})^2 = 9 - 2 = 7$

Thus, the required quadratic equation is

$x^2 - Sx + P = 0$

$x^2 - 6x + 7 = 0$

2. If α, β are the roots of the equation $x^2 - 3x + 6 = 0$

Form equations whose roots are

a) $2\alpha + 1, 2\beta + 1$

$x^2 - 3x + 6 = 0$

Here a=1, b= − 3, c=6

Let α, β be the roots of the given equation

$$\text{Then} \quad \alpha + \beta = -\frac{b}{a} \quad and \quad \alpha\beta = \frac{c}{a}$$

$$= -\frac{(-3)}{1} \qquad \alpha\beta = \frac{6}{1}$$

$$= 3 \qquad\qquad = 6$$

Let roots of the new equation be $2\alpha + 1, 2\beta + 1$

S = Sum of roots $\qquad$ P = Product of roots

$$\begin{aligned} &= (2\alpha+1)+(2\beta+1) && = (2\alpha+1)(2\beta+1) \\ &= 2\alpha+1+2\beta+1 && = (2\alpha+1)(2\beta+1) \\ &= 2\alpha+2\beta+2 && = 4\alpha\beta+2\alpha+2\beta+1 \\ &= 2(\alpha+\beta)+2 && = 4(6)+2(3)+1 \\ &= 2(3)+2 && = 24+6+1 \\ &= 8 && = 31 \end{aligned}$$

Thus, the required quadratic equation is

$x^2 - Sx + P = 0$

$x^2 - 8x + 31 = 0$

b) α^2, β^2

Solution:

$x^2 - 3x + 6 = 0$

Here a=1, b= − 3, c=6

Let α, β be the roots of the given equation

$$\text{Then} \quad \alpha + \beta = -\frac{b}{a} \quad and \quad \alpha\beta = \frac{c}{a}$$

$$= -\frac{(-3)}{1} \qquad \alpha\beta = \frac{6}{1}$$

$$= 3 \qquad\qquad = 6$$

Let roots of the new equation be α^2, β^2

S = Sum of roots P = Product of roots

$= \alpha^2 + \beta^2$ $= (\alpha^2)(\beta^2)$

$= (\alpha + \beta)^2 - 2\alpha\beta$ $= (\alpha^2 \beta^2)$

$= (3)^2 - 2(6)$ $= (\alpha\beta)^2$

$= 9 - 12$ $= (6)^2$

$= -3$ $= 36$

Thus, the required quadratic equation is

$x^2 - Sx + P = 0$

$x^2 - (-3)x + 36 = 0$

$x^2 + 3x + 36 = 0$

c) $\dfrac{1}{\alpha}, \dfrac{1}{\beta}$

Solution:

$x^2 - 3x + 6 = 0$

Here a=1, b= – 3, c=6

Let α, β be the roots of the given equation

Then $\alpha + \beta = -\dfrac{b}{a}$ *and* $\alpha\beta = \dfrac{c}{a}$

$= -\dfrac{(-3)}{1}$ $\alpha\beta = \dfrac{6}{1}$

$= 3$ $= 6$

Let roots of the new equation be $\dfrac{1}{\alpha}, \dfrac{1}{\beta}$

S = Sum of roots P = Product of roots

$= \dfrac{1}{\alpha} + \dfrac{1}{\beta}$ $= \left(\dfrac{1}{\alpha}\right)\left(\dfrac{1}{\beta}\right)$

$= \dfrac{\alpha + \beta}{\alpha\beta}$ $= \dfrac{1}{\alpha\beta}$

Thus, the required quadratic equation is

$x^2 - Sx + P = 0$

$x^2 - \frac{1}{2}x + \frac{1}{6} = 0$

$\Rightarrow 6x^2 - 3x + 1 = 0$

d) $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$

Solution:

$x^2 - 3x + 6 = 0$

Here a=1, b= − 3, c=6

Let α, β be the roots of the given equation

Then $\alpha + \beta = -\frac{b}{a}$ *and* $\alpha\beta = \frac{c}{a}$

$= -\frac{(-3)}{1}$ $\alpha\beta = \frac{6}{1}$

$= 3$ $= 6$

Let roots of the new equation be $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$

S = Sum of roots P = Product of roots

$= \frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ $= \left(\frac{\alpha}{\beta}\right)\left(\frac{\beta}{\alpha}\right)$

$= \frac{\alpha^2 + \beta^2}{\alpha\beta}$ $= 1$

$= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$

$= \frac{(3)^2 - 2(6)}{6}$

$= \frac{9 - 12}{6}$

$= -\frac{3}{6}$

$= -\frac{1}{2}$

$$x^2 - Sx + P = 0$$

$$x^2 - \left(-\frac{1}{2}\right)x + 1 = 0$$

$$\Rightarrow 2x^2 + x + 2 = 0$$

e) $\alpha + \beta, \frac{1}{\alpha} + \frac{1}{\beta}$

Solution:

$x^2 - 3x + 6 = 0$

Here a=1, b= − 3, c=6

Let α, β be the roots of the given equation

Then $\alpha + \beta = -\frac{b}{a}$ *and* $\alpha\beta = \frac{c}{a}$

$$= -\frac{(-3)}{1} \qquad \alpha\beta = \frac{6}{1}$$

$$= 3 \qquad = 6$$

Let roots of the new equation be $\alpha + \beta, \frac{1}{\alpha} + \frac{1}{\beta}$

S = Sum of roots and P = Product of roots

$$S = \alpha + \beta + \frac{1}{\alpha} + \frac{1}{\beta} \qquad P = (\alpha + \beta)\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$$

$$= (\alpha + \beta) + \frac{\alpha + \beta}{\alpha\beta} \qquad = (\alpha + \beta)\left(\frac{\alpha + \beta}{\alpha\beta}\right)$$

$$= 3 + \frac{3}{6} \qquad = (3)\left(\frac{3}{6}\right)$$

$$= 3 + \frac{1}{2} \qquad = \frac{3}{2}$$

$$= \frac{7}{2}$$

Thus, the required quadratic equation is

$$x^2 - Sx + P = 0$$

$$x^2 - \frac{7}{2}x + \frac{3}{2} = 0$$

3. If α, β are the roots of the equation $x^2 + px + q = 0$

Form equations whose roots are

a) α^2, β^2

Solution:

$x^2 + px + q = 0$

Here $a = 1,\ b = p,\ c = q$

Let α, β be the roots of the given equation

$$\text{Then} \quad \alpha + \beta = -\frac{b}{a} \quad \textit{and} \quad \alpha\beta = \frac{c}{a}$$

$$= -\frac{p}{1} \qquad \alpha\beta = \frac{q}{1}$$

$$= -p \qquad = q$$

Let roots of the new equation be α^2, β^2

$$S = \text{Sum of roots} \quad \text{and} \quad P = \text{Product of roots}$$

$$= \alpha^2 + \beta^2 \qquad = (\alpha^2)(\beta^2)$$

$$= (\alpha + \beta)^2 - 2\alpha\beta \qquad = (\alpha\beta)^2$$

$$= (-p)^2 - 2q \qquad = q^2$$

$$= p^2 - 2q$$

Thus, the required quadratic equation is

$x^2 - Sx + P = 0$

$x^2 - (p^2 - 2q)x + q^2 = 0$

b) $\dfrac{\alpha}{\beta}, \dfrac{\beta}{\alpha}$

Solution:

$x^2 + px + q = 0$

Here $a = 1,\ b = p,\ c = q$

Let α, β be the roots of the given equation

$$= -\frac{p}{1} \qquad \alpha\beta = \frac{q}{1}$$

$$= -p \qquad = q$$

Let roots of the new equation be $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$

S = Sum of roots $\qquad$ P = Product of roots

$$= \frac{\alpha}{\beta} + \frac{\beta}{\alpha} \qquad = \left(\frac{\alpha}{\beta}\right)\left(\frac{\beta}{\alpha}\right)$$

$$= \frac{\alpha^2 + \beta^2}{\alpha\beta} \qquad = 1$$

$$= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$$

$$= \frac{(-p)^2 - 2q}{q}$$

$$= \frac{p^2 - 2q}{q}$$

Thus, the required quadratic equation is

$$x^2 - Sx + P = 0$$

$$x^2 - \left(\frac{p^2 - 2q}{q}\right)x + 1 = 0$$

$$qx^2 - (p^2 - 2q)x + q = 0$$