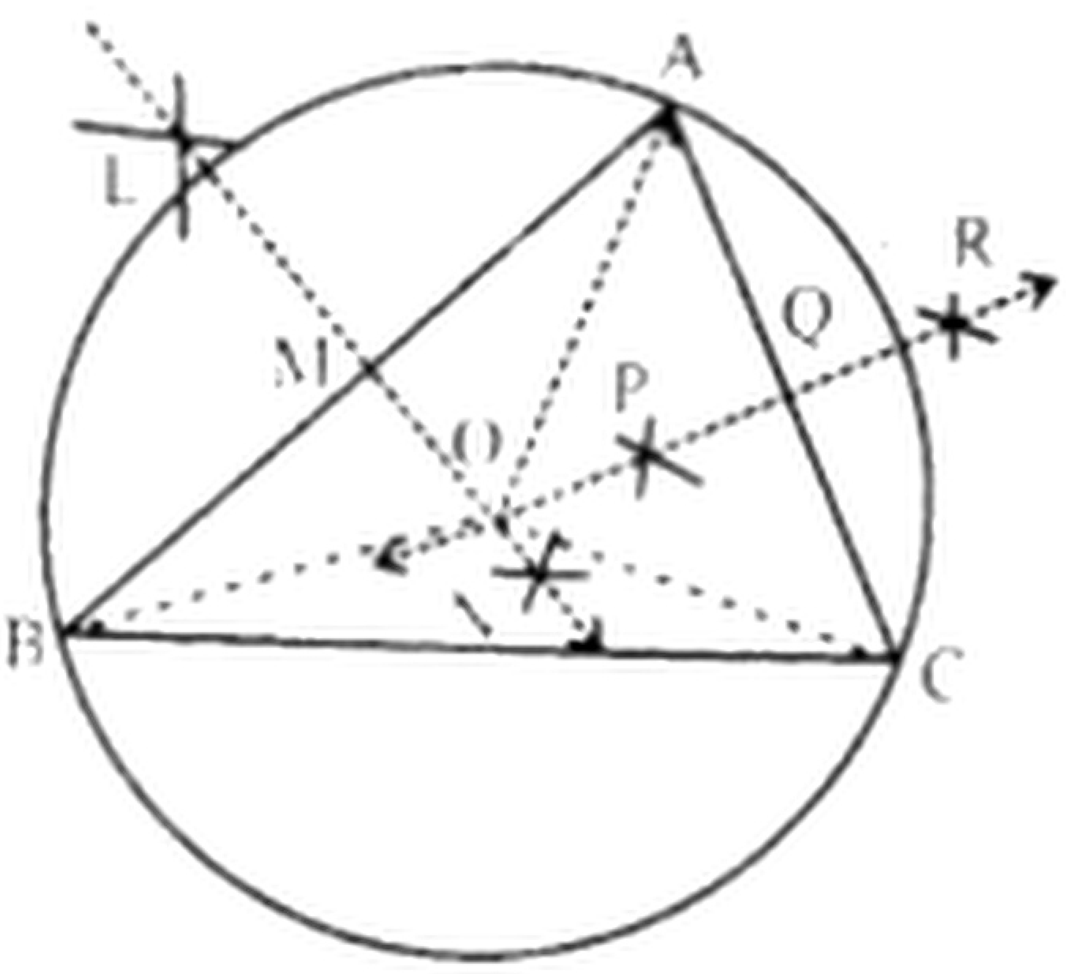
(i) Circumscribe a circle about a given triangle:



Given:

Triangle ABC.

Steps of Construction:

1. Draw $\overleftrightarrow{LMN}$ as perpendicular bisector of side $\overline{AB}$.
2. Draw $\overleftrightarrow{PQR}$ as perpendicular bisector of side $\overline{AC}$.
3. $\overleftrightarrow{LN}$ and $\overleftrightarrow{PP}$ intersect at point O.
4. With centre O and radius

   $m\overline{OA} = m\overline{OB} = m\overline{OC}$, draw a circle.

This circle will pass through A, B and C whereas O is the circumcenter of the circumscribed circle.

Remember:

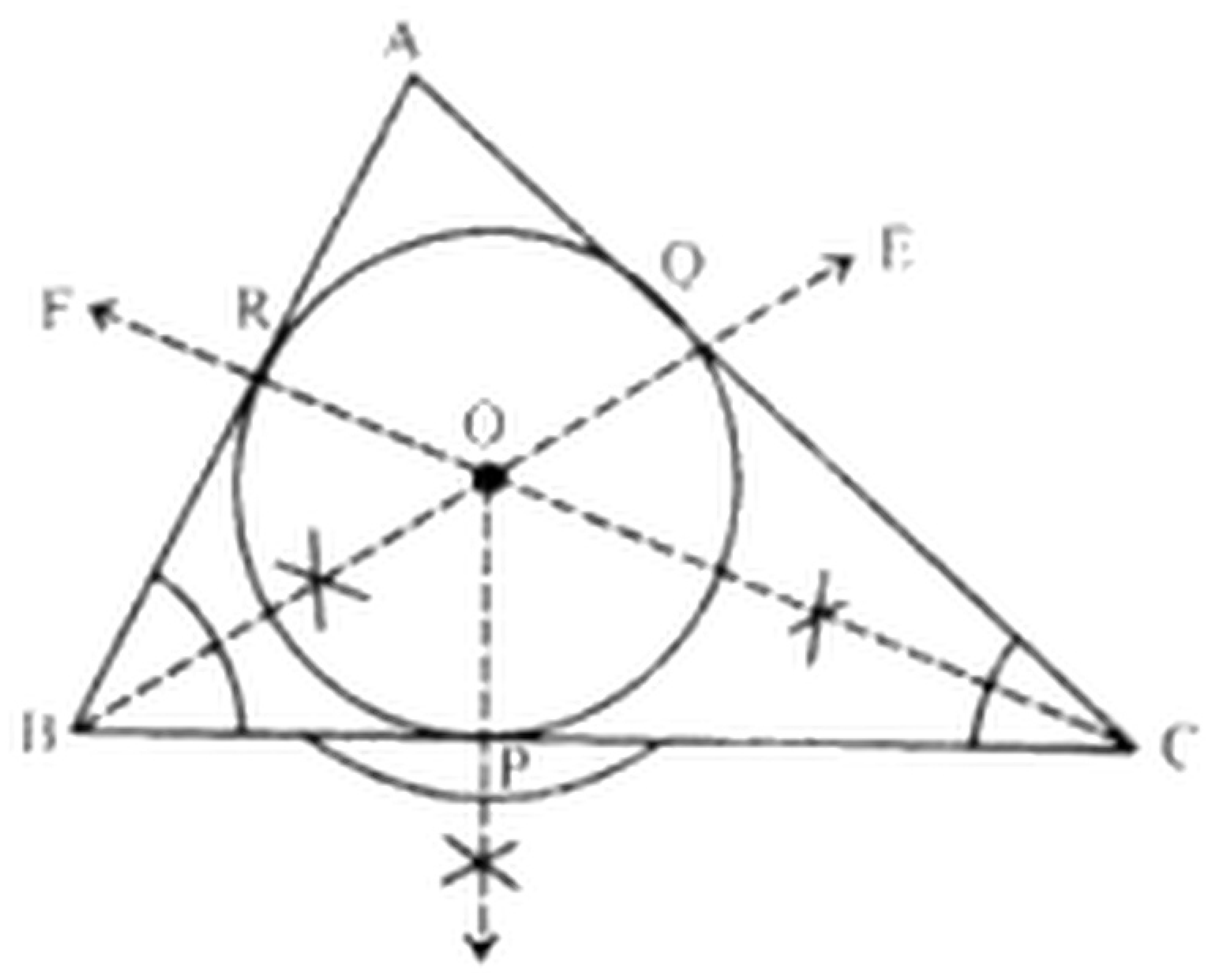
The circle passing through, the vertices of triangle ABC is known as **circumcircle**, its radius as **circumradius** and centre as **circumcenter.**

(ii) Inscribe a circle in a given triangle:

Given:

A triangle ABC.

Steps of Construction:



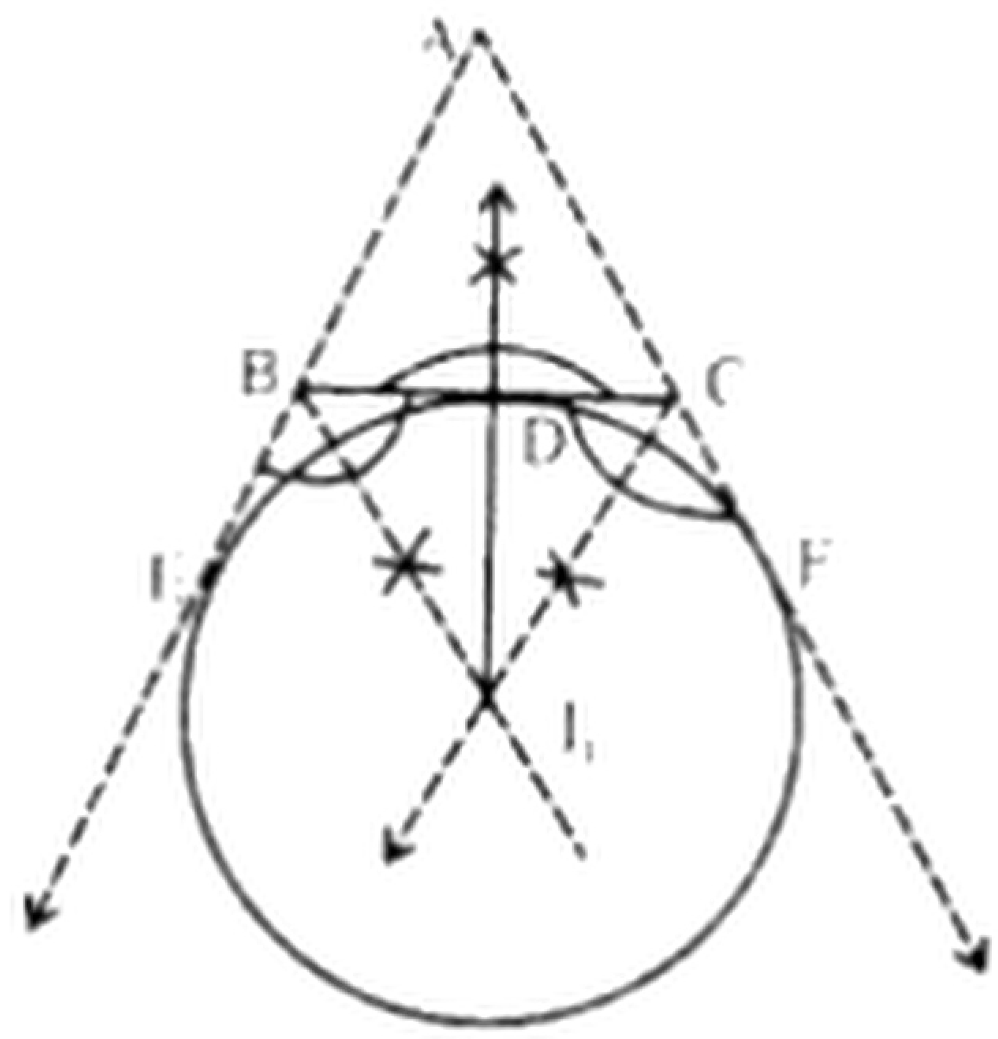
1. Draw $\overrightarrow{BE}$ and $\overrightarrow{CF}$ to bisect the angles ABC and ACB respectively. Rays $\overrightarrow{BE}$ and $\overrightarrow{CF}$ intersect each other at point O.

2. O is the centre of the inscribed circle.

3. From O draw $\overrightarrow{OP}$ perpendicular to $\overline{BC}$. With centre O and radius $\overline{OP}$ draw a circle. This circle is the inscribed circle of triangle ABC.

Remember:

A circle which touches the three sides of a triangle internally is known as **incircle**, its radius as **inradius** and centre as **in centre**.



(iii) Escribe a circle to a given triangle:

Given:

A triangle ABC

Steps of Construction:

1. Produce the sides $\overline{AB}$ and $\overline{AC}$ of ΔABC.

2. Draw bisectors of exterior angles ABC and ACB.

   These bisectors of exterior angles meet at I_1.

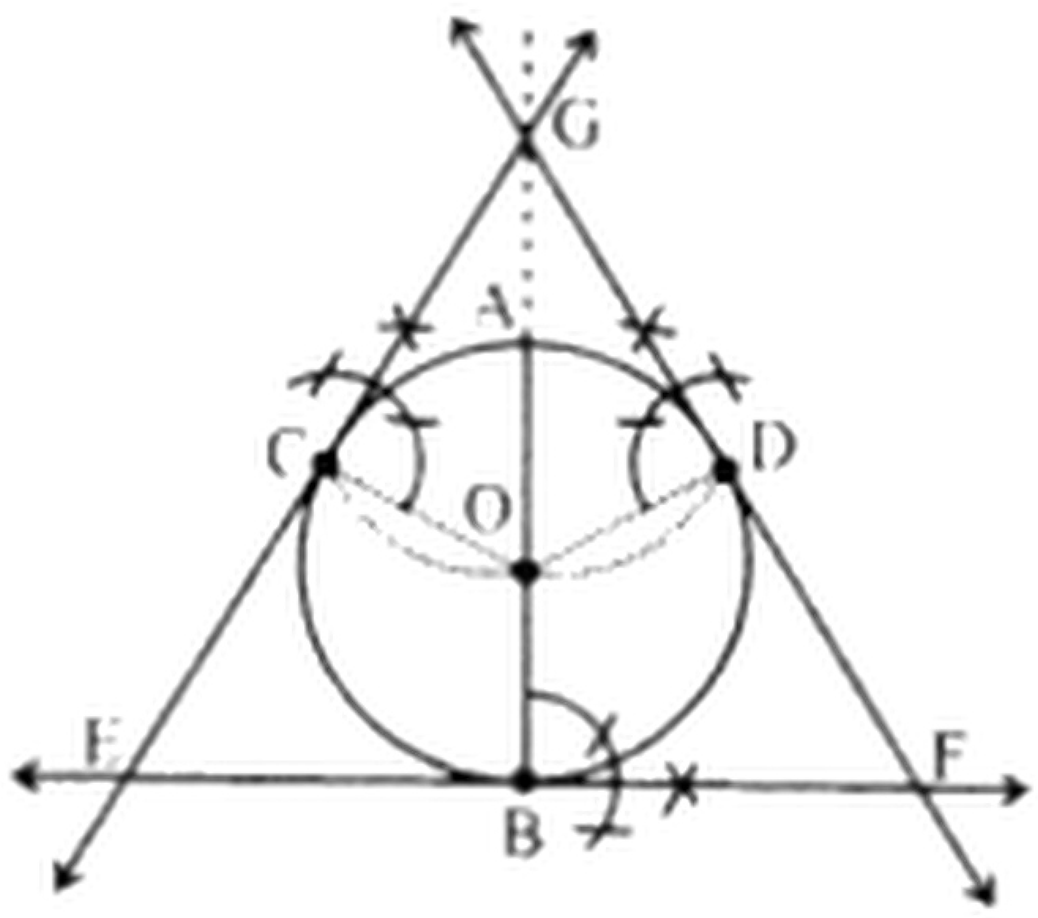
3. From I_1 draw perpendicular on side $\overline{BC}$ of ΔABC. Which I_1D intersect BC at D. I_1D is the radius of the escribed circle with centre at I_1.

4. Draw the circle with radius I_1D and centre at I_1 that will touch the side BC of the ΔABC externally and the produced sides AB and AC.

Escribed circle:

The circle touching one side of the triangle externally and two produced sides internally is called escribed circle (e-circle). The centre of e-circle is called **e-centre** and radius is called **e-radius**.

(iv) Circumscribe an equilateral triangle about a given circle



Given:

A circle with centre O of reasonable radius.

Steps of Construction:

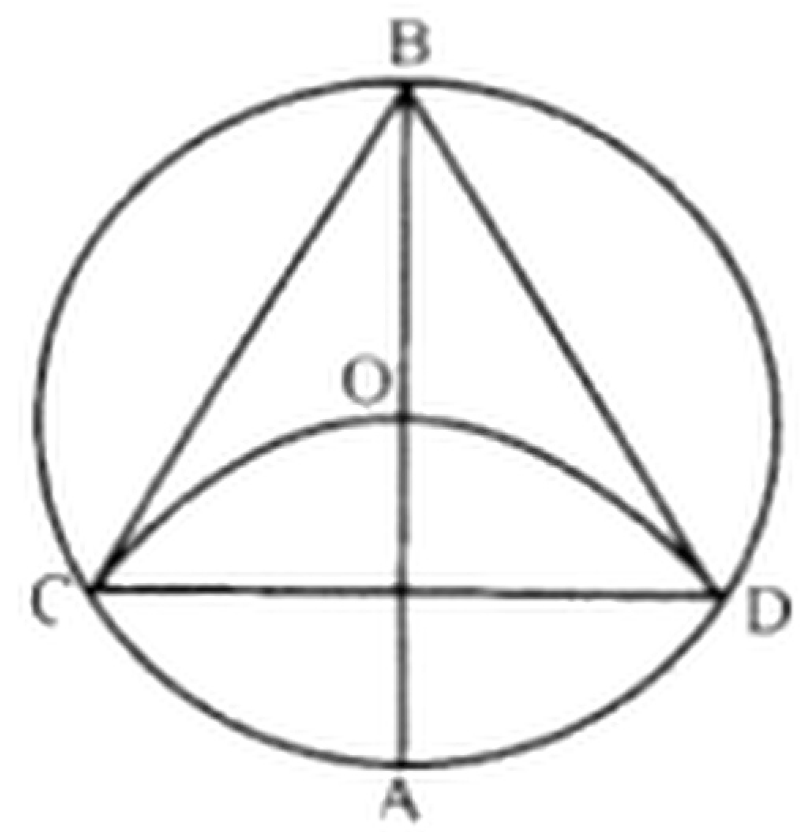
1 Draw $\overline{AB}$, the diameter of the circle, for locating.

2. Draw an arc of radius $m\overline{OA}$ with centre at A for locating points C and D on the circle.

3. Join O to the points C and D.

4. Draw tangents to the circle at points B, C and D.

5. These tangents intersect at points E, F and G.



(v) Inscribe an equilateral triangle in a given circle.

Given:

A circle with centre at O.

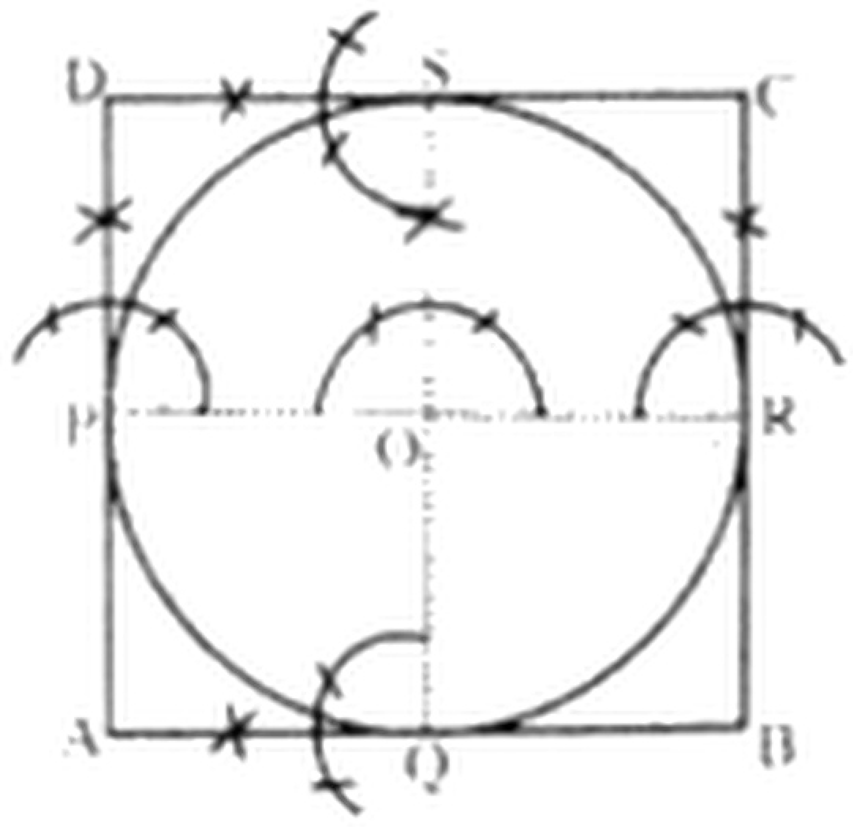
Steps of Construction:

1. Draw any diameter $\overline{AB}$ of the circle.

2. Draw an arc of radius OA from point A. The arc cuts the circle at points C and D.

3. Join the prints B, C and D to form straight line segments $\overline{BC}$, $\overline{CD}$ and $\overline{BD}$.
   Triangle BCD is the required inscribed equilateral triangle.

(vi) Circumscribe a square about a given circle:



Given:

A circle with centre at O.

Steps of Construction:

1. Draw two diameters $\overline{PR}$ and $\overline{QS}$ which bisect each other at right angle.
2. At points P, Q, R and S draw tangents to the circle.
3. Produce the tangents to meet each other at A, B, C and D.
   ABCD is the required circumscribed square.

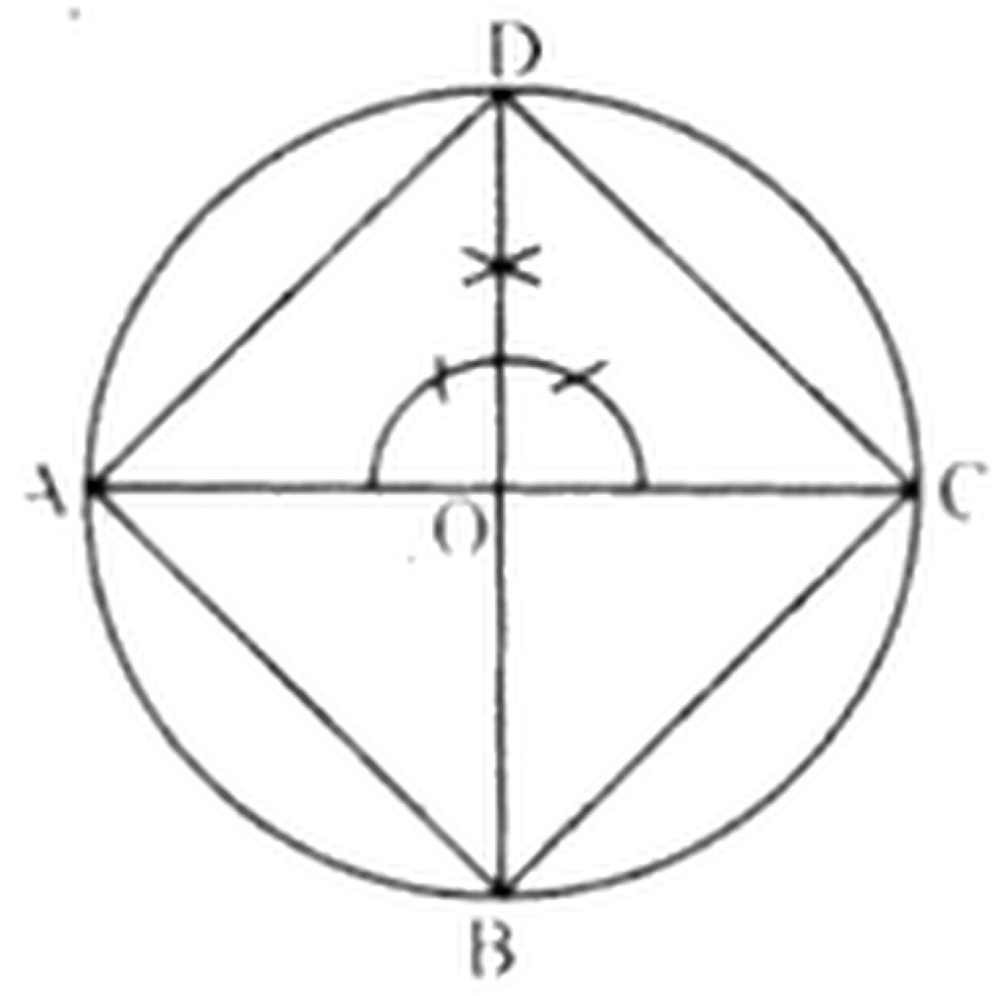
(vii) Inscribe a square in a given circle:

Given:

A circle, with centre at O.

Steps of Construction:

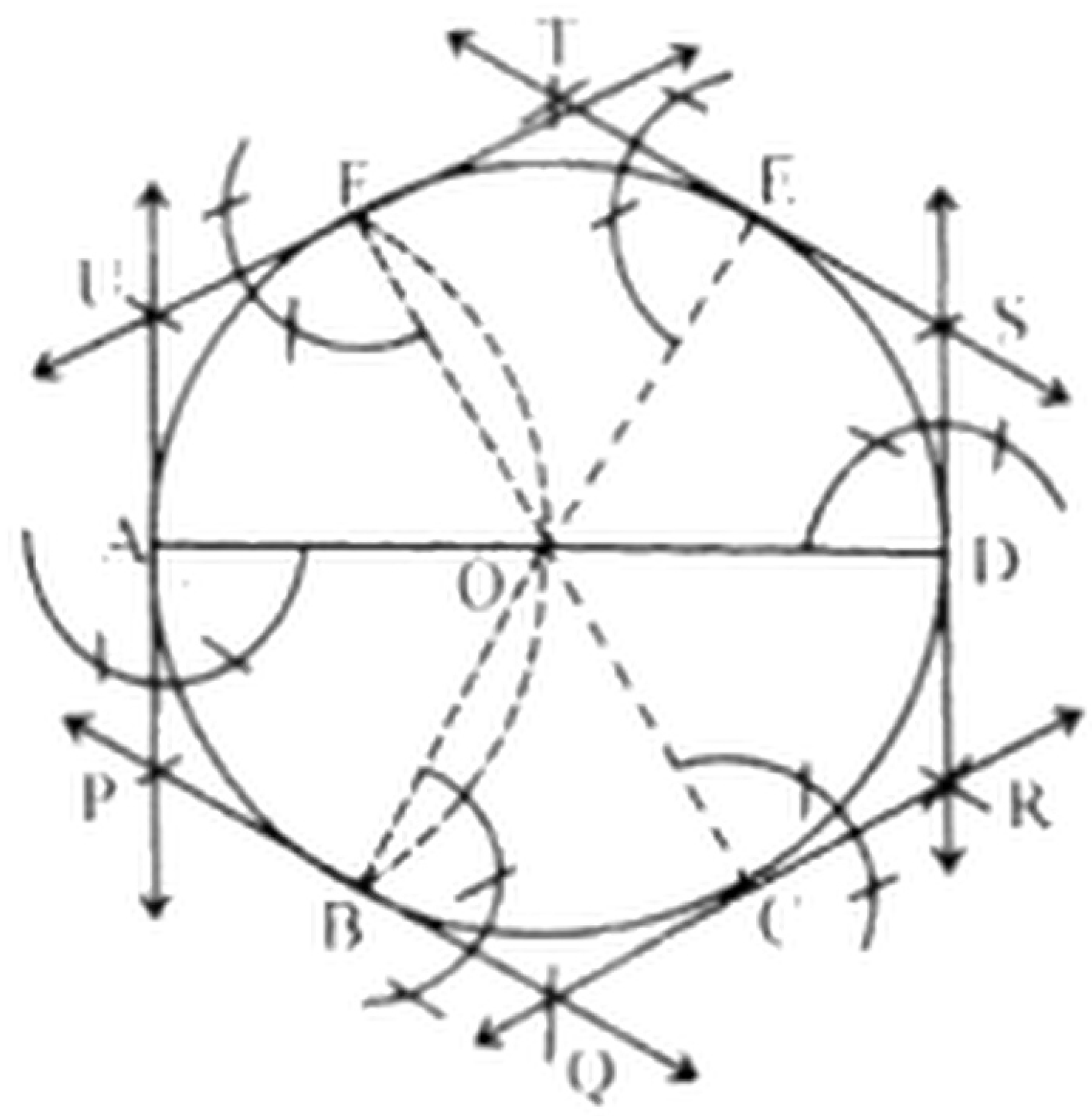
1. Through O draw two diameters AC and BD which bisect each other at right angle.
2. Join A with B, B with C, C with D and D with A.
   ABCD is the required square inscribed in the circle.



(viii) Circumscribe a regular hexagon about a given circle

Given:

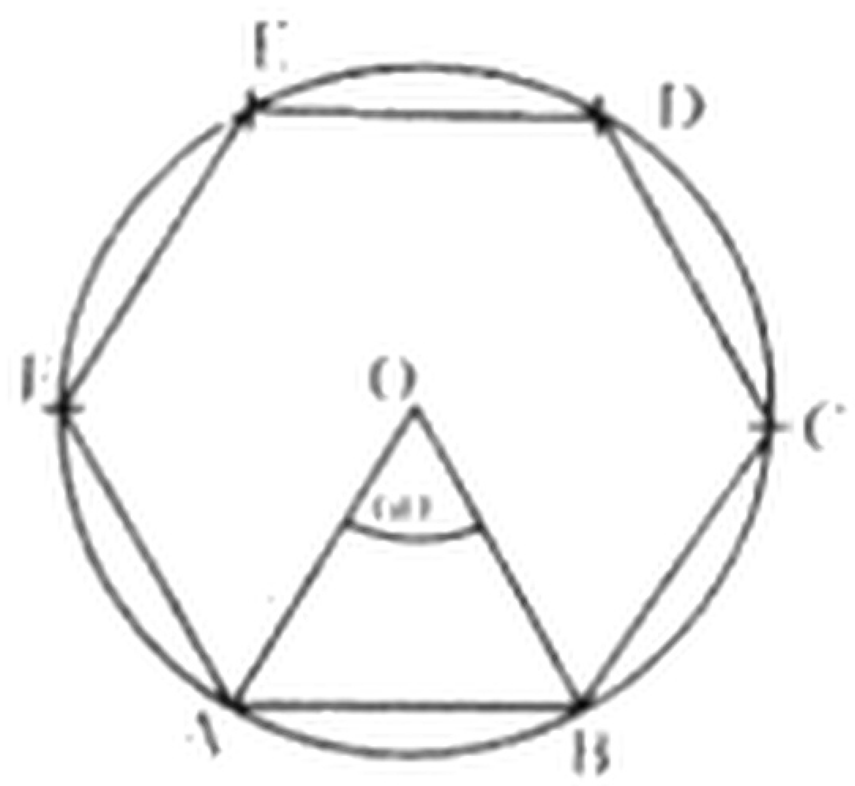
A circle with centre at O.

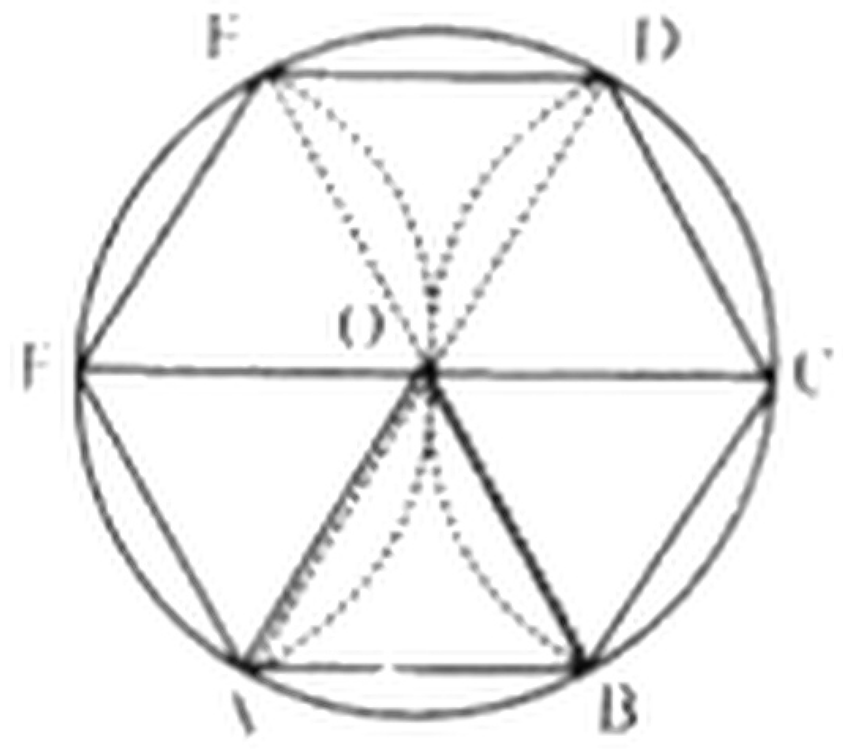


Steps of Construction:

1. Draw any diameter $\overline{AD}$.
2. From point A draw an arc of radius $\overline{AO}$ (the radius of the circle), which cuts the circle at points B and F.
3. Join B with O and extend it to meet the circle at E.
4. Join F with O and extend it to meet the circle at C.
5. Draw tangents to the circle at points A, B, C, D, E and F intersecting one another at points P, Q, R, S, T and U respectively.
6. Thus, PQRSTU is the circumscribed regular hexagon.

(ix) Inscribe a regular hexagon in a given circle





Given:

A circle, with centre at O.

Steps of Construction:

1. Take any point A on the circle and point with O.

2. From point A, draw an arc of radius $\overline{OA}$ which intersects the circle at point B and F.

3. Join O and A with points B and F.

4. ΔOAB and ΔOAF are equilateral therefore $\angle AOB$ and $\angle AOF$ are of measure $60°$ i.e., $m\overline{OA} = m\overline{AB} = m\overline{AF}$.

5. Produce $\overline{FO}$ to meet the circle at C. Join B to C. Since in $\angle BOC = 60°$ therefore $m\overline{BC} = m\overline{OA}$.

6. From C and F, draw arcs of radius $\overline{OA}$, which intersect the circle at points D and E.

7. Join C to D, D to E and E to F ultimately. We have

   $m\overline{OA} = m\overline{OB} = m\overline{OC} = m\overline{OD} = m\overline{OE} = m\overline{OF}$

Thus, the figure ABCDEF is a regular hexagon inscribed in the circle.

Exercise 13.2

1. Circumscribe a circle about a triangle ABC with sides

$|\overline{AB}| = 6$ cm , $|\overline{BC}| = 3$ cm , $|\overline{CA}| = 4$ cm

Also measure its circum radius.

Solution:

Given:

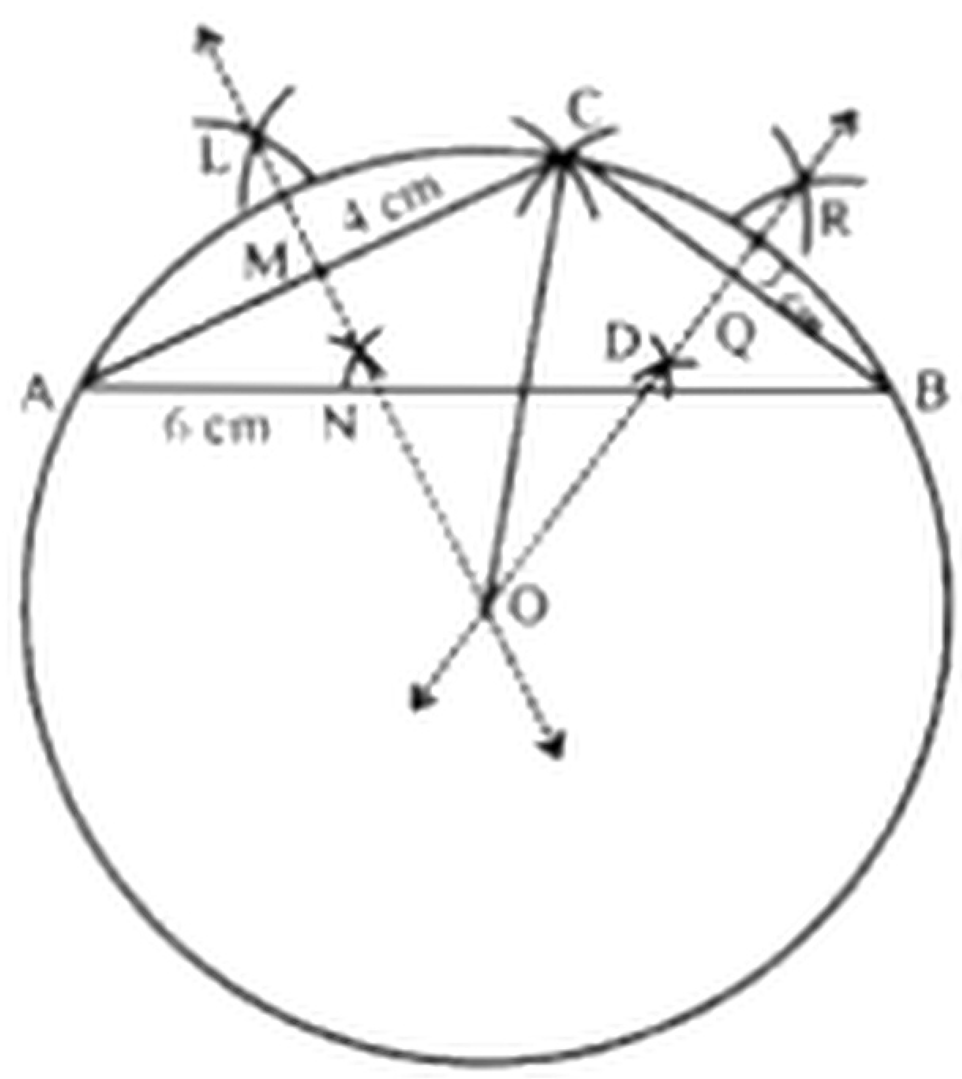
Three sides

$|\overline{AB}| = 6$ cm , $|\overline{BC}| = 3$ cm , $|\overline{CA}| = 4$ cm

Required:

To construct a circumscribed circle about a triangle using given information.

Steps of Construction:



1. Draw a line segment $|AB| = 6$ cm
2. With centre at A, draw an arc of radius 4 cm.
3. With centre at B, draw an arc of radius 3 cm which cuts the previous arc at point C.
4. Join C with A and B.
5. Thus ABC is the required triangle.
6. Draw $\overline{LMN}$ as perpendicular bisector of side $\overline{AB}$.
7. Draw $\overleftrightarrow{PQR}$ as perpendicular bisector of side $\overline{BC}$.
8. $\overleftrightarrow{LN}$ and $\overleftrightarrow{PR}$ intersect at point O.

9. With centre O and radius $m\overline{OA} = m\overline{OB} = m\overline{OC}$, draw a circle.

10. This circle will pass through A, B and C whereas O is circum center of the circumscribed circle

    Here $m\overline{OA} = m\overline{OB} = m\overline{OC} = 3.3$ cm.

2. Inscribe a circle in a triangle ABC with sides.

$|\overline{AB}| = 5$ cm, $|\overline{BC}| = 3$ cm, $|\overline{CA}| = 3$ cm. Also measure its in-radius.

Solution:

Given:

Three sides

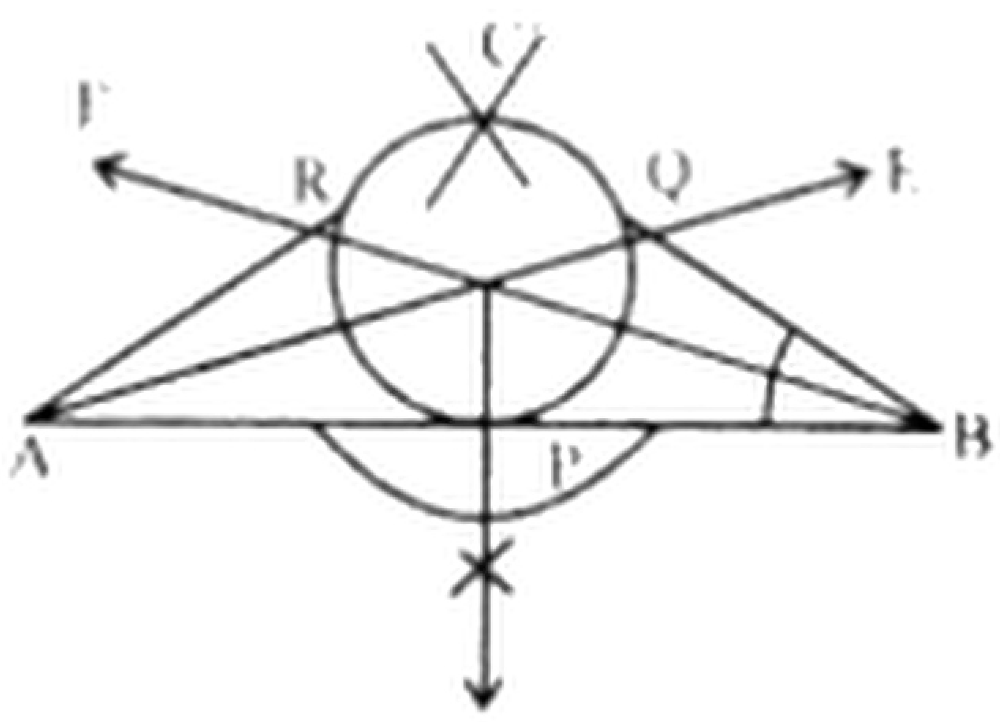
$|\overline{AB}| = 5$ cm, $|\overline{BC}| = 3$ cm, $|\overline{CA}| = 3$ cm

Required:

To construct an inscribed circle about a triangle using given information.

Steps of Construction:

1. Draw a line segment $|\overline{AB}| = 5$cm
2. With centre at A, draw an arc of radius 3 cm.
3. With centre at B, draw an arc of radius 3 cm which cuts the previous arc at point C.



4. Join C with A and B.
5. Thus, ABC is the required triangle.
6. Draw $\overrightarrow{AE}$ and $\overrightarrow{BF}$ to bisect the angles BAC and ABC. Rays $\overrightarrow{AE}$ and $\overrightarrow{BF}$ intersect each other at point O
7. O is the centre of the inscribed circle.

8. From O draw $\overline{OP}$ perpendicular to $\overline{AB}$.

9. With centre O and radius $\overline{OP}$ draw a circle.

10. This circle is the inscribed circle of triangle ABC.

Here $m\overline{OP} = m\overline{OQ} = m\overline{OR} = 1$ cm (approximately).

3. Escribe a circle opposite to vertex A to a triangle ABC with sides

$|\overline{AB}| = 6$ cm, $|\overline{BC}| = 4$ cm, $|\overline{CA}| = 3$ cm. Find its radius also.

Solution:

Given:

Three sides

$|\overline{AB}| = 6$ cm, $|\overline{BC}| = 4$ cm, $|\overline{CA}| = 3$ cm

Required:

To construct an escribed circle opposite to vertex A to a triangle using given information.

Steps of Construction:

1. Draw a line segment $|\overline{BC}| = 4$ cm.

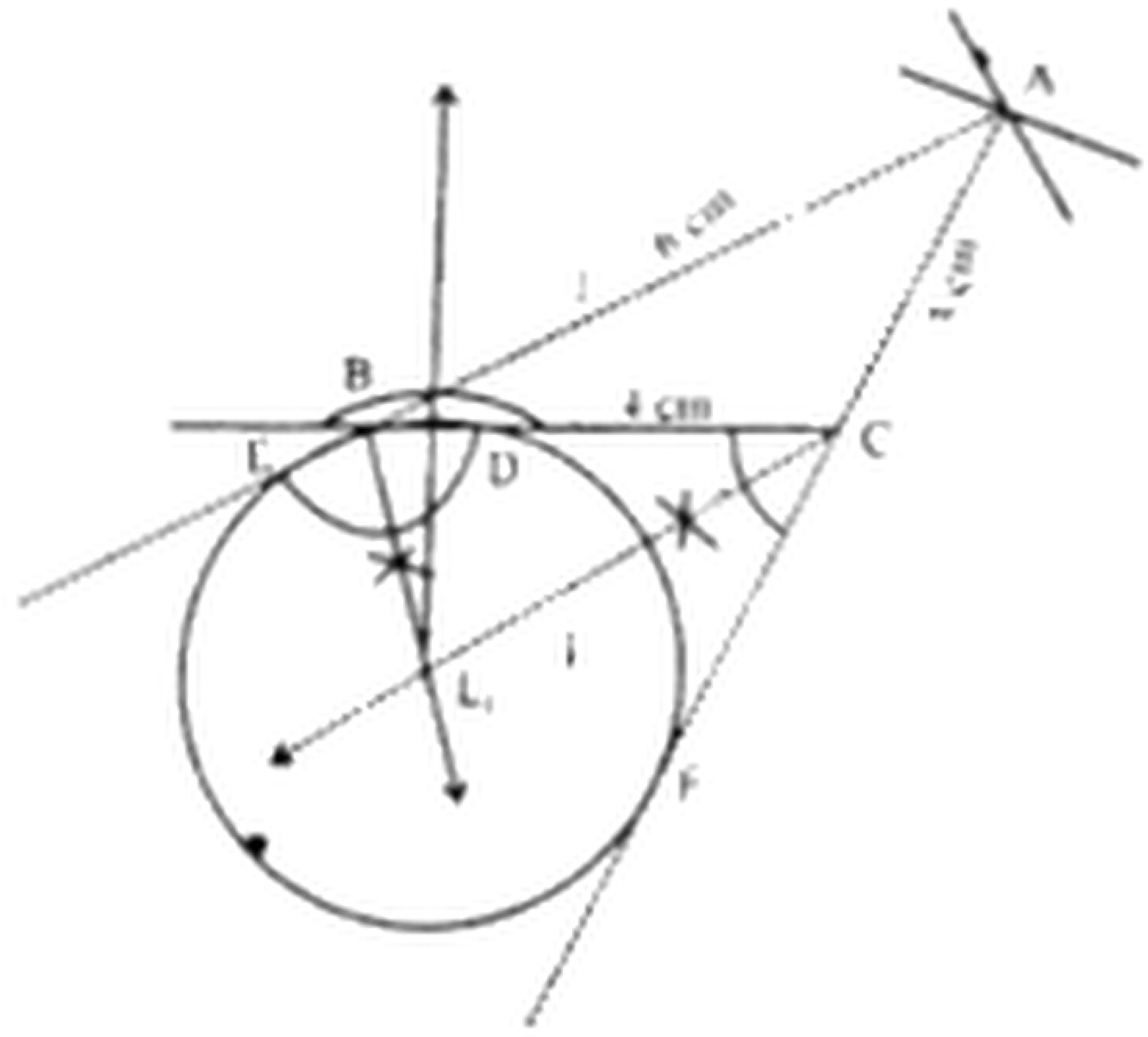
2. With centre at B, draw an arc of 6 cm.

3. With centre at C, draw an arc of 3cm which cuts

the previous arc at point A.

4. Join A with B and C.
5. Thus, ABC is the required triangle.
6. Produce the sides AB and BC of ΔABC.
7. Draw bisectors of exterior angles ABC and ACB.

   These bisectors of exterior angles meet at L.
8. From L, draw perpendicular to side BC of ΔABC which intersects BC at D. LD is the radius of the escribed circle with centre at L.
9. Draw the circle with radius LD and centre at L, that will touch the side BC of the ΔABC externally and the produced sides AB and AC internally.



4. Circumscribe a circle about an equilateral triangle ABC with each side of length 4 cm.

Solution:

Given:

Equilateral triangle ABC with each side of length 4cm.

Required:

To construct a circumscribed circle about an equilateral triangle using given information.

Steps of Construction:

1. Draw a line segment $|\overline{AB}| = 4$ cm
2. With centre at A, draw an arc of radius 4 cm.
3. With centre at B, draw an arc of radius 4 cm which cuts the previous arc

at point C.

4. Join C with A and B.

5. Thus $\overline{ABC}$ is the required triangle.

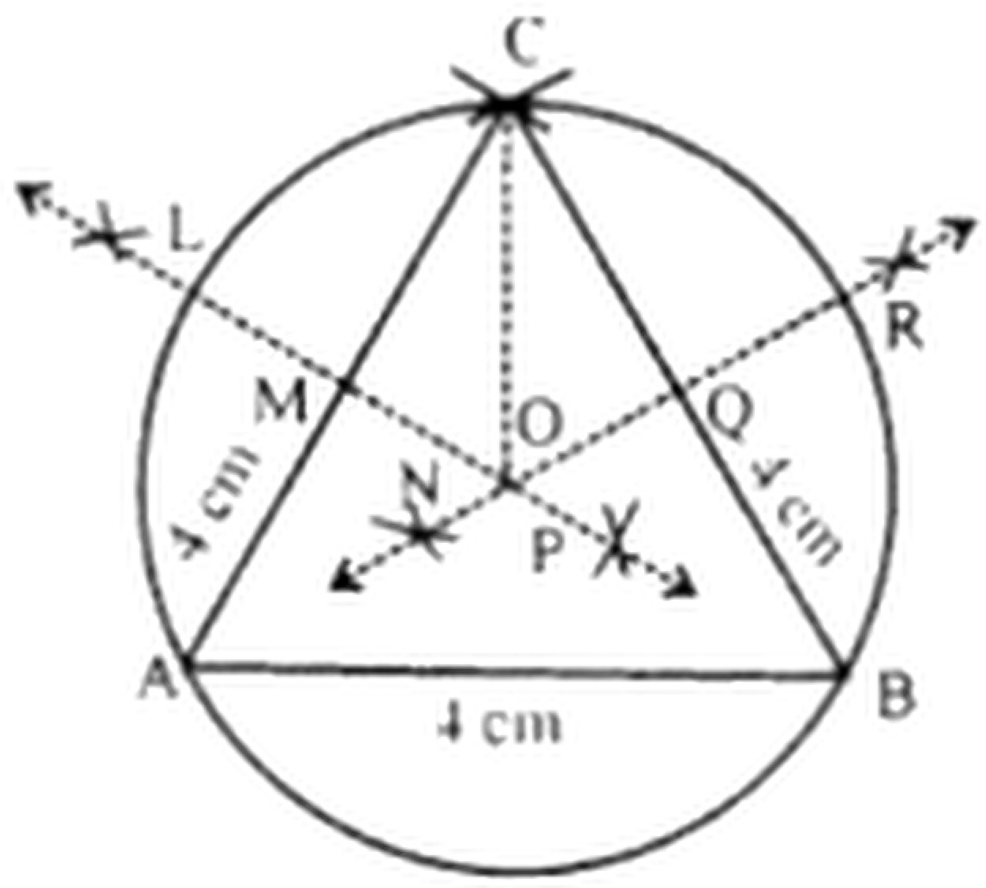
6. Draw $\overline{LMN}$ as perpendicular bisector to of side AC.

7. Draw $\overline{PQR}$ as perpendicular bisector of side $\overline{BC}$.

8. $\overline{LN}$ and $\overline{PQ}$ intersect at point O.

9. With centre at O and radius $m\overline{OA} = m\overline{OB} = m\overline{OC}$, draw a circle.

10. This circle will pass through A, B and C whereas O is circumcenter of the circumscribed circle.



5. Inscribe a circle in an equilateral triangle ABC with each side of length 5 cm.

Solution:

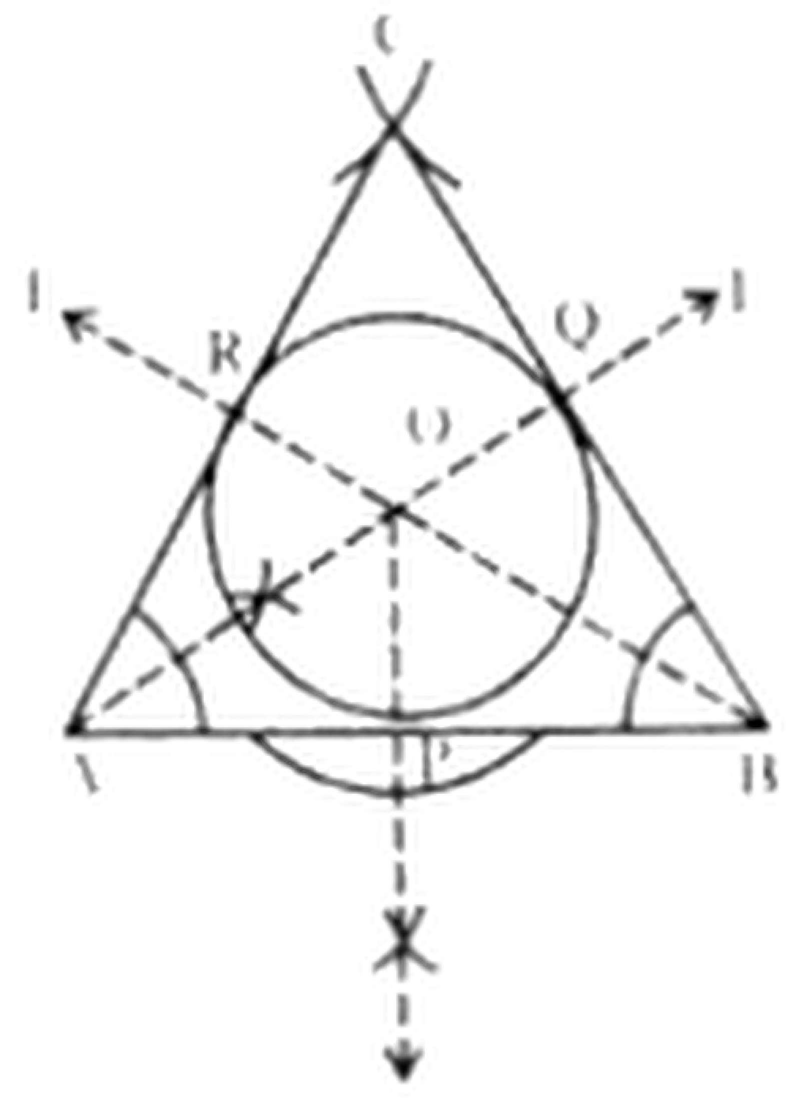
Given:

Equilateral triangle ABC with each side of length 5 cm.

Required:

To construct an inscribed circle about a triangle using given information.

Steps of Construction:



1. Draw a line segment $|\overline{AB}| = 5cm$
2. With centre at A, draw an arc of radius 5cm.
3. With centre at B, draw an arc of radius 5cm which cuts the previous arc at point C.
4. Join C with A and B.
5. Thus ABC is the required triangle.
6. Draw $\overrightarrow{AE}$ and $\overrightarrow{BF}$ to bisect the angles BAC and ABC. Rays $\overrightarrow{AE}$ and $\overrightarrow{BF}$ intersect each other at point O
7. O is the centre of the inscribed circle.
8. From O draw $\overrightarrow{OP}$ perpendicular to $\overline{AB}$.
9. With centre O and radius $\overline{OP}$ draw a circle.
10. This circle is the inscribed circle of triangle ABC.

6. Circumscribe and inscribe circles with regard to a right-angle triangle with sides, 3 cm, 4 cm and 5 cm.

Solution:

Given:

Three sides

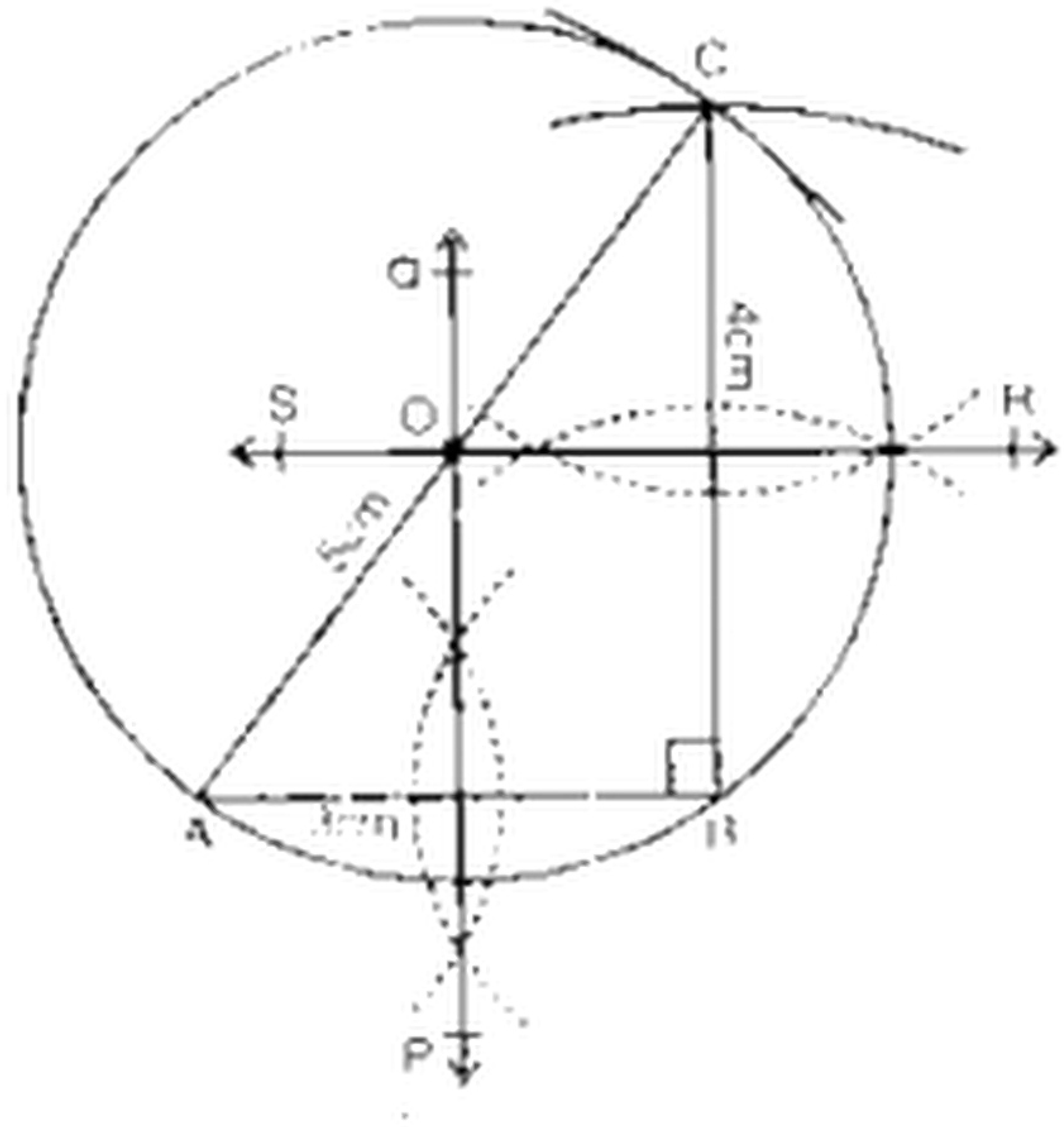
$|\overline{AB}| = 3$ cm, $|\overline{BC}| = 4$ cm, $|\overline{CA}| = 5$ cm.

Required:

To construct an inscribed and circumscribed circle about a triangle using given information.

(i) Circumcircle

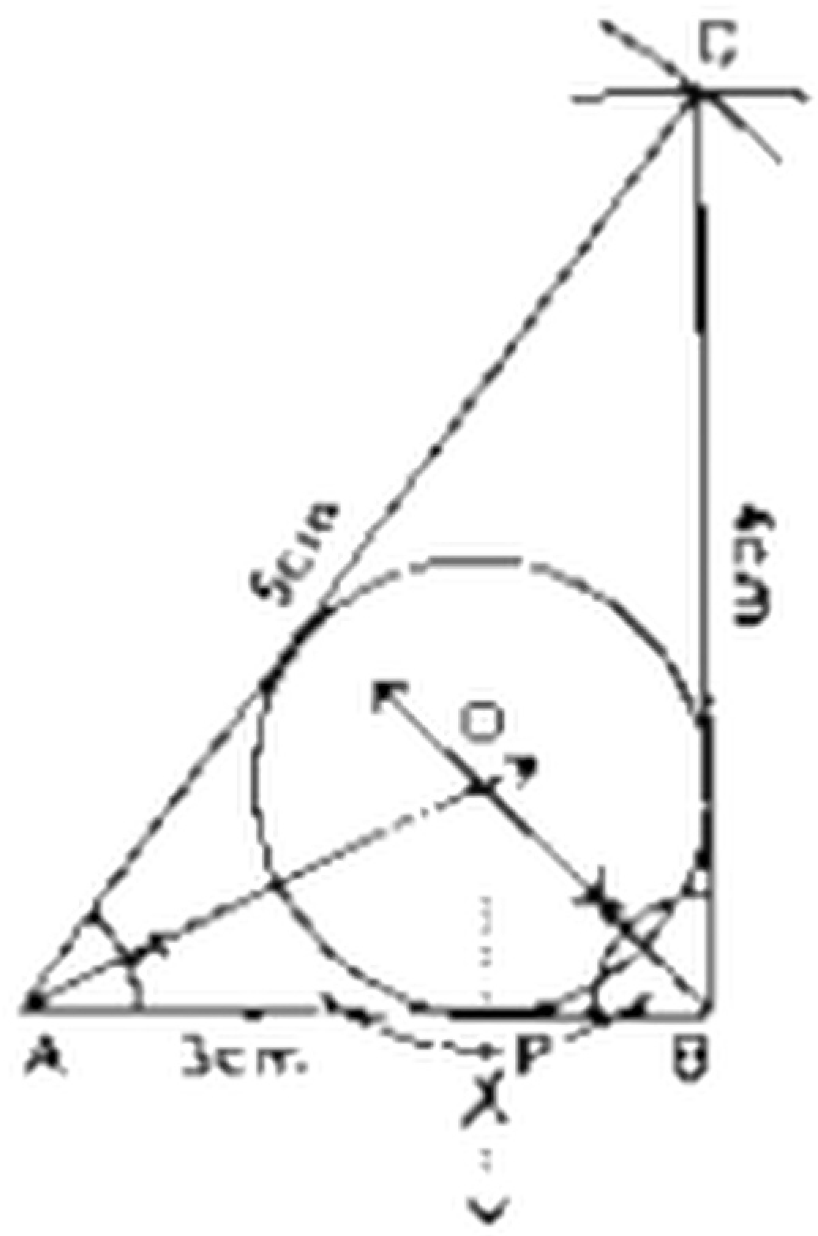
Steps of Construction:



1. Construct a right-angle triangle ABC with sides 3 cm, 4 cm and 5 cm.
2. Draw right bisectors $\overleftrightarrow{PQ}$ and $\overleftrightarrow{RS}$ of each side $\overline{AB}$ and $\overline{BC}$ respectively intersecting each other at point O.
3. Taking O as centre and radius equal to m$\overline{OA}$ or m$\overline{OB}$ or m$\overline{OC}$, we draw a circle passing trough the points A, B and C.
4. This is our required circumcircle whose radius is measured to be 2.5 cm.

(ii) Inscribed Circle

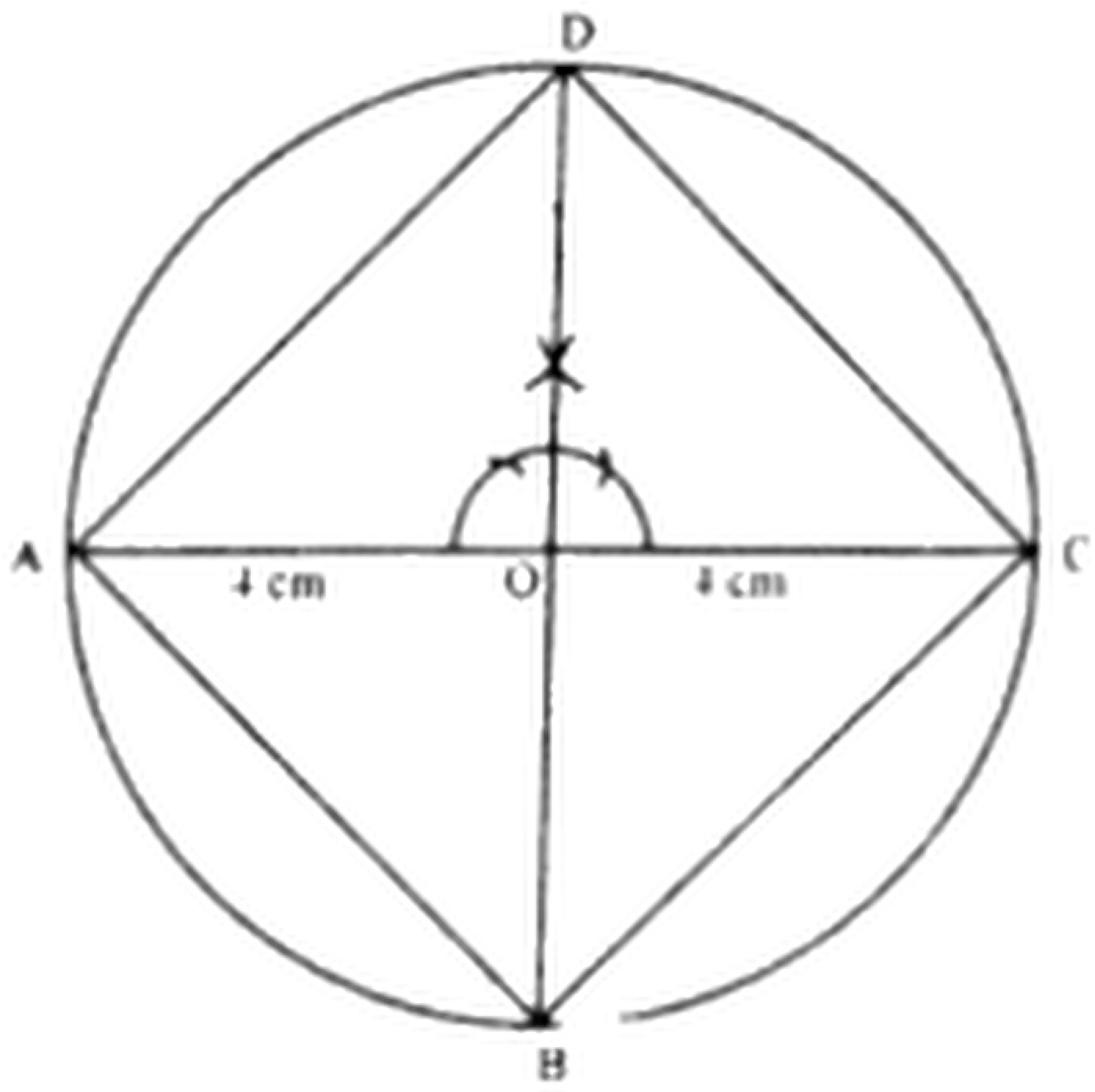
Steps of Construction:



1. Construct a right-angle triangle ABC according to the given conditions.
2. We draw the bisectors of ∠A and ∠B intersecting each other at point O.
3. From point O, we draw $\overline{OP}$ perpendicular to $\overline{AB}$.
4. Taking O as centre and radius equal to $\overline{OP}$, we draw a circle, touching three sides of triangle internally.
5. This is the required incircle whose radius is measured to be 1 cm.

7. In and around the circle of radius 4 cm draw a square.

(i) In a circle of radius 4 cm draw a square:



Solution:

Given:

A circle of radius 4cm.

Required:

Draw a square inside the circle.

Steps of Construction:

1. Draw circle of radius 4cm with O as a centre.
2. Through O draw two diameters $\overline{AC}$ and $\overline{BD}$ which bisect each other at right angle.
3. Join A with B, B with C, C with D and D with A.
4. Thus, ABCD is the required square inscribed in the circle.

(ii) Around a circle of radius 4 cm draw a square:

Solution:

Given:

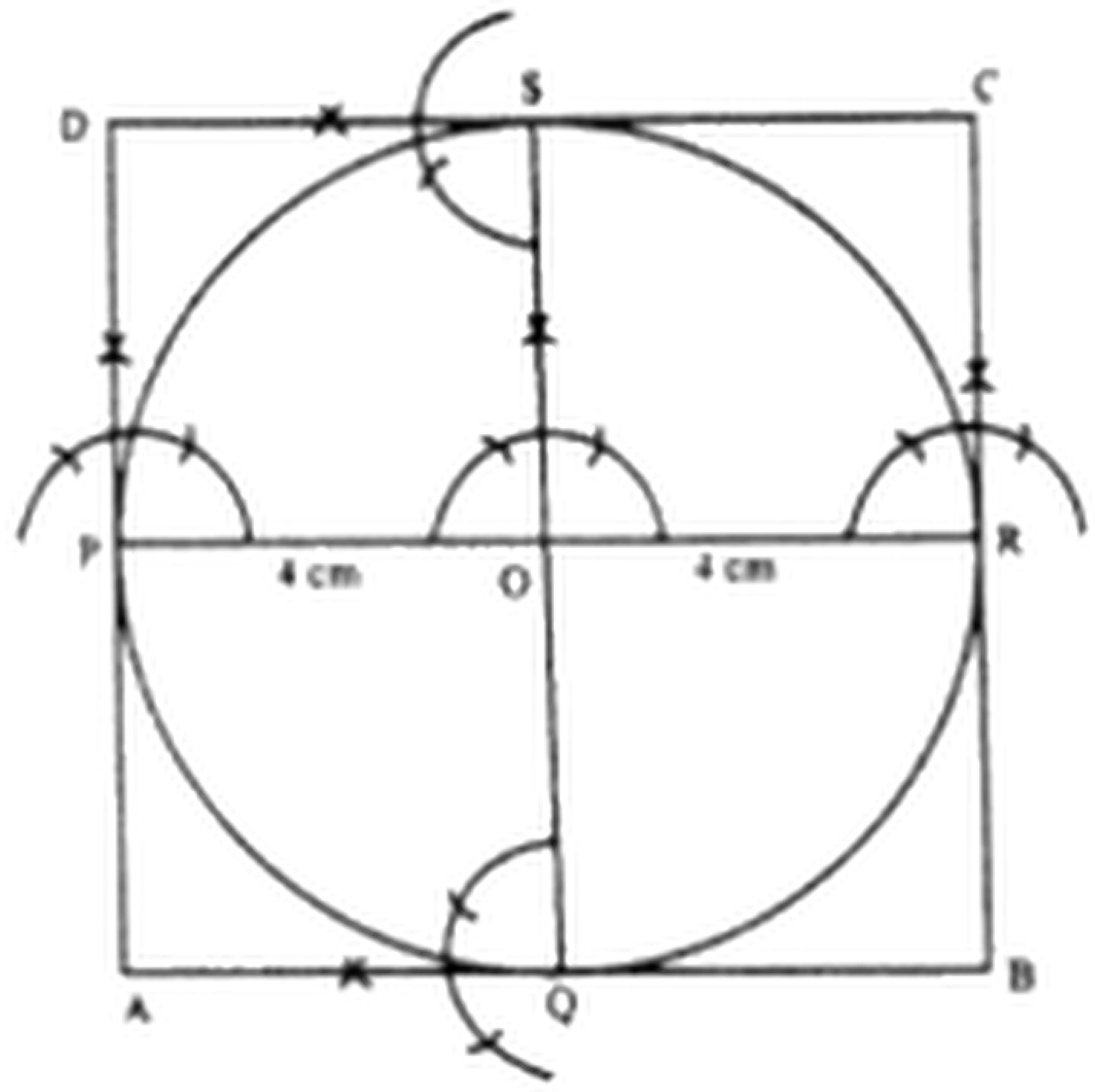
A circle of radius 4cm.

Required:

Draw a square around the circle.

Steps of Construction:

1. Draw circle of radius 4cm with O as a centre.
2. Draw two diameters $\overline{PR}$ and $\overline{QS}$ which bisect each other at right angle.
3. At point P, Q, R and S draw tangents to meet each other at A, B, C and D. ABCD is the required circumscribed square.



8. In and around the circle of radius 3.5 cm draw a regular hexagon.

(i) In a circle of radius 3.5 cm draw a hexagon:

Solution:

Given:

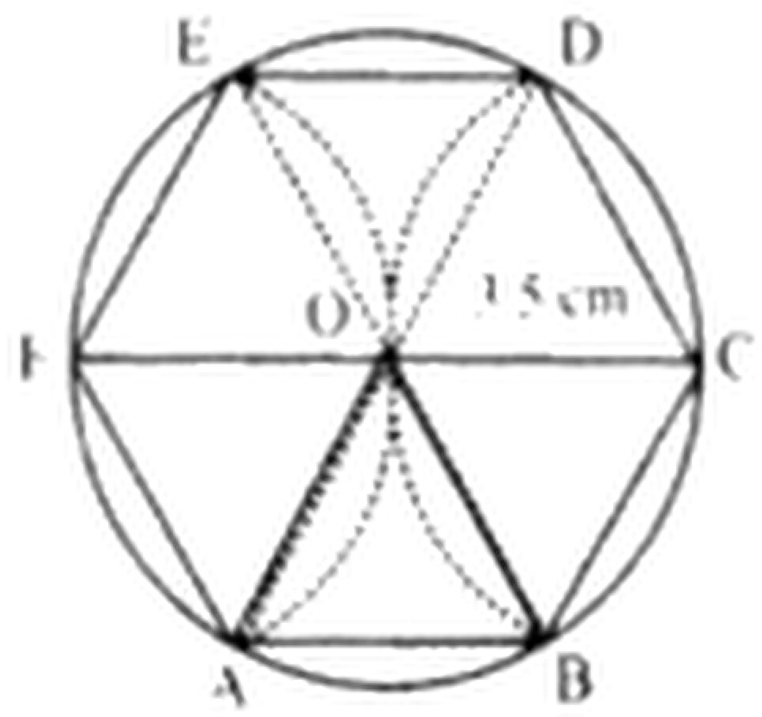
A circle of radius 3.5cm.

Required:

Draw a regular hexagon inside the circle.

Steps of Construction:

1. Take any point O.
2. Take O as centre of and draw a circle of radius 3.5 cm.
3. Take any point A on the circumference of the circle.
4. From point A, draw an arc of radius $\overline{OA}$ which intersects the circle at point B and F.
5. Join O and A with points B and F.



6. ΔOAB and ΔOAF are equilateral triangles therefore $\angle AOB$ and $\angle AOF$ are of measure 60° i.e., $m\overline{OA} = m\overline{AB} = m\overline{AF}$.

7. Produce $\overline{FO}$ to meet the circle at C. Join B to C, Since in $\angle BOC = 60°$ therefore $m\overline{BC} = m\overline{OA}$.

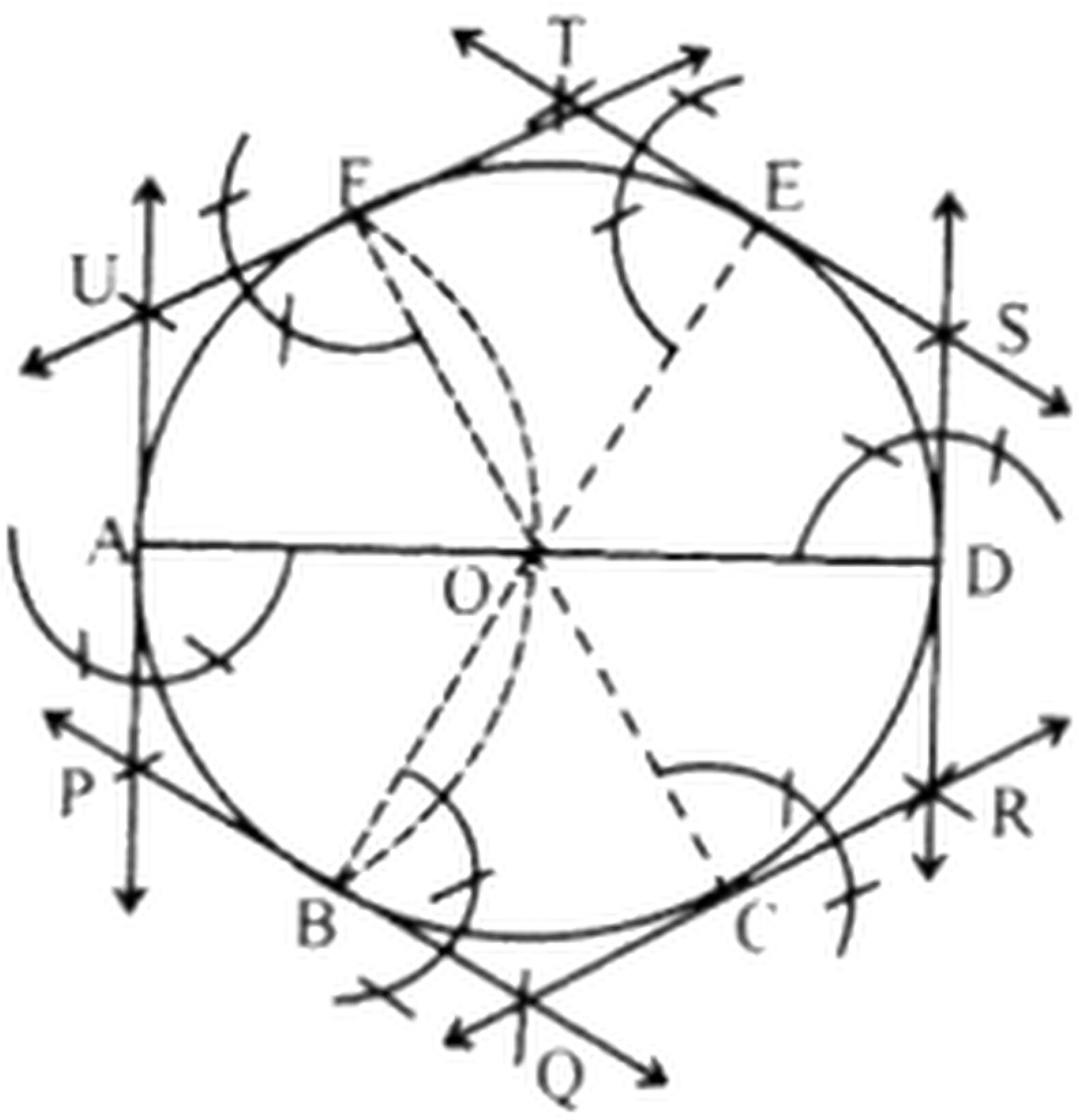
8. From C and F, draw arcs of radius $\overline{OA}$, which intersect the circle at points D and E.

9. Join C to D, D to E and E to F. Ultimately we have

   $m\overline{OA} = m\overline{OB} = m\overline{OC} = m\overline{OD} = m\overline{OE} = m\overline{OF}$

   Thus, the figure ABCDEF is a regular hexagon inscribed in the circle.

(ii) Around a circle of radius 3.5 cm draw a regular hexagon:



Given:

A circle of radius 3.5 cm.

Steps of Construction:

1. Draw a diameter $\overline{AD}$ = 7 cm.
2. From point A draw an arc of radius $\overline{AO}$ = 3.5 cm(the radius of the circle), which cuts the circle at points B and F.
3. Join B with O and extend it to meet the circle at E.
4. Join F with O and extend it to meet the circle at C.
5. Draw tangents to the circle at points A, B, C, D, E and F intersecting one another at points P, Q, R, S, T and U respectively.
6. Thus, PQRSTU is the circumscribed regular hexagon.

9. Circumscribe a regular hexagon about a circle of radius 3cm.

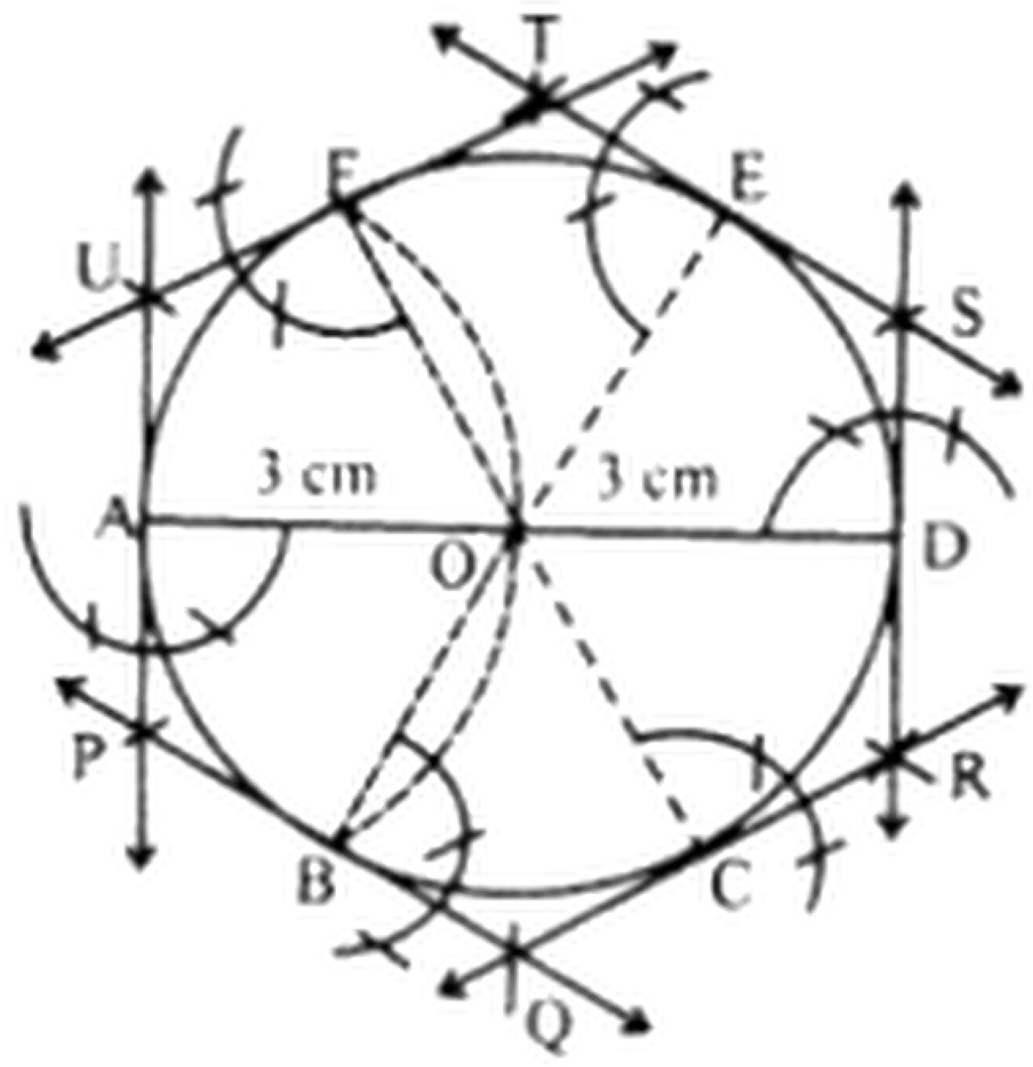
Given:

A circle of radius 3 cm.

Steps of Construction:

1. Draw a diameter $\overline{AD}$ = 6 cm.

2. From point A draw an arc of radius $\overline{AO}$ = 3 cm (the radius of the circle), which cuts the circle at points B and F.



3. Join B with O and extend it to meet the circle at E.

4. Join F with O and extend it to meet the circle at C.

5. Draw tangents to the circle at points A, B, C, D, E and F intersecting one another at points P, Q, R, S, T and U respectively.

6. Thus, PQRSTU is the circumscribed regular hexagon.