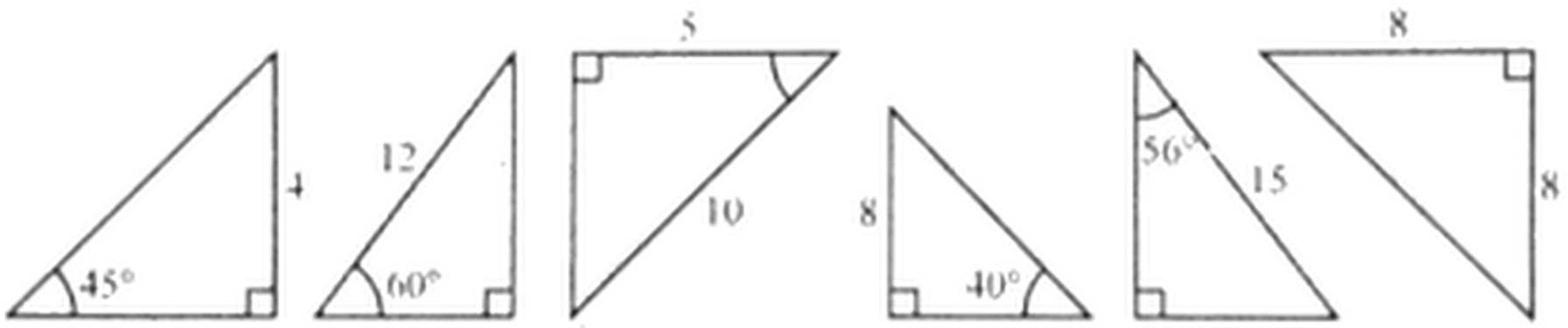
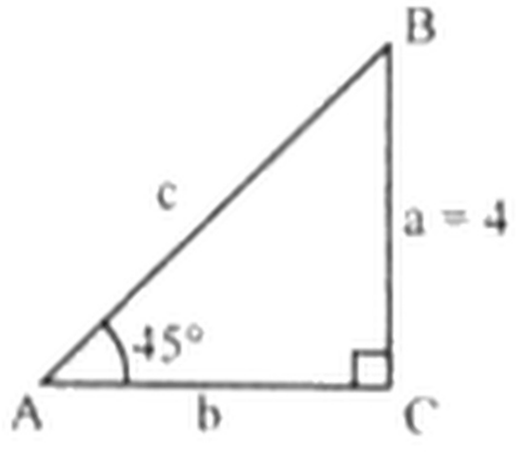
Exercise 12.2

1. Find the unknown angles and sides of the following triangles:



i. ∠A = 45°, ∠C = 90°, ∠B=?



Solution

we know that

$\angle A + \angle B + \angle C = 180°$

$\Rightarrow \angle B = 180° - 90° - 45° = 45°$

and a = 4. b = ? c = ?

$\sin 45° = \frac{a}{c}$

$\Rightarrow c = \frac{a}{\sin 45°} = \frac{4}{0.707} = 5.65$

$\Rightarrow c = 5.65$

$\tan 45° = \frac{a}{b}$

$$1=\frac{a}{b}$$

b=a = 4

$\Rightarrow$b=4

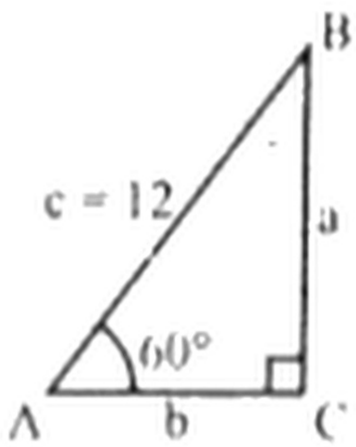
Hence

$\angle$A = 450 a = 4

$\angle$B = 45° b = 4

$\angle$C = 90° c =5.67

ii. $\angle$A = 60°, $\angle$C= 90°, $\angle$B = ?



Solution

we know that

$\angle$A+ $\angle$B+ $\angle$C = 180°

$\Rightarrow$ $\angle$B = 180°-90° -60°

$\angle$B = 30°

and a=?, b = ?, c= 12

$$\sin 60° = \frac{a}{c}$$

a = c sin 60°

$$\Rightarrow a= 12\frac{\sqrt{3}}{2}=6\sqrt{3}$$

$$\cos 60°=\frac{b}{c}$$

$b = c \cos 60°$

$= 12 \times \frac{1}{2} = 6$

$\Rightarrow b = 6$

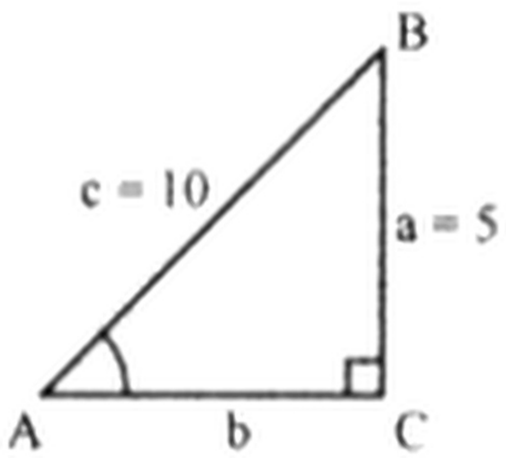
Hence

$\angle A = 60°$ $a = 6\sqrt{3}$

$\angle B = 30°$ $b = 6$

$\angle C = 90°$ $c = 12$

iii. c = 10, a = 5, b = ?



Solution

We know that by Pythagoras Theorem

$c^2 = a^2 + b^2$

$\Rightarrow b = \sqrt{c^2 - a^2}$

$\sqrt{10^2 - 5^2} = \sqrt{100 - 25} = \sqrt{75} = 5\sqrt{3}$

$\Rightarrow b = 5\sqrt{3}$

$\angle C = 90°$, $\angle A = ?$, $\angle B = ?$

$\text{Sin}(\angle A) = \frac{a}{c} = \frac{5}{10}$

$\Rightarrow \angle A = \sin^{-1}(\frac{1}{2}) = 30°$

$\Rightarrow \angle A = 180° - \angle A - \angle C$

$= 180° - 30° - 90° = 60°$

$\Rightarrow \angle B = 60°$

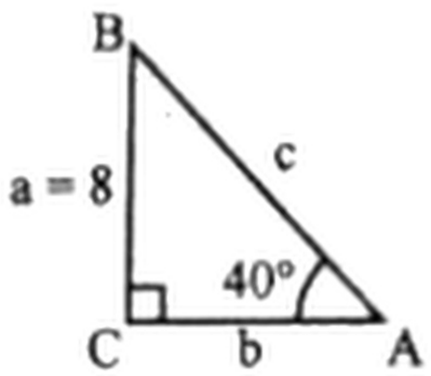
Hence

$\angle A = 30°$ a=5

$\angle B = 60°$ $b = 5\sqrt{3}$

$\angle C = 90°$ c=10

iv. $\angle A=40°$, $\angle B=?$, $\angle C = 90°$



Solution

we know that

$\angle A + \angle B + \angle C = 180°$

$\angle B = 180° - \angle C - \angle A$

$\angle B = 180° - 90° - 40° = 50°$

$\Rightarrow \angle B = 50°$

and a = 8, b=?, c= ?

$$\sin A = \frac{a}{c} = \frac{8}{c}$$

$$\Rightarrow c = \frac{8}{\tan(40^\circ)} = \frac{8}{0.642} = 12.445$$

$$\tan(A) = \frac{a}{b} = \frac{8}{b}$$

$$b = \frac{8}{\sin(40^\circ)} = \frac{8}{0.8391} = 9.534$$

$\Rightarrow b=9.534$

Hence

$\angle A = 40°$ $\quad a=8$

$\angle B = 50°$ $\quad b = 9.534$

$\angle C = 90°$ $\quad c = 12.445$

v. $\angle A = 56°$, $\angle B = ?$, $\angle C = 90°$

Solution

we know that

$\angle A + \angle B + \angle C = 180°$

$\Rightarrow \angle B = 180°-90° - 56 = 34°$

$\Rightarrow \angle B = 34°$

$\sin A = \frac{a}{c}$

$\Rightarrow a = c \sin 56°$

$= 15(0.829) = 12.435$

$a = 12.435$

$\cos 56° = \frac{b}{c}$

$\Rightarrow b= c \cos 56°$

$= 15(0.56) =8.388$

$b = 8.388$

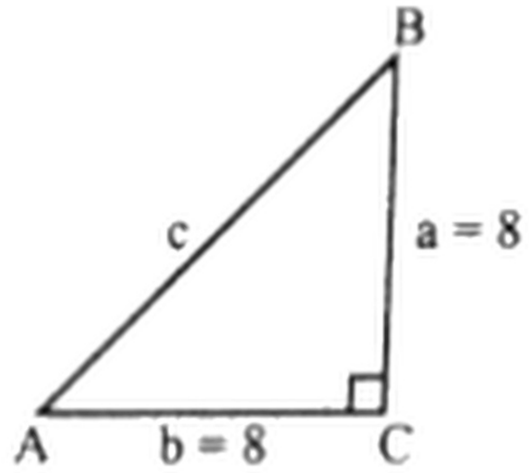
Hence

$\angle A = 56°$ $a = 12.435$

$\angle B = 340$ $b = 8.388$

$\angle C = 90°$ $c = 15$

vi. a=8, b = 8, c = ?



Solution

We know that by Pythagoras theorem.

$c^2 = a^2 + b^2$

$= 8^2 + 8^2 = 64 + 64 = 128$

$\Rightarrow c^2 = 128$

$c = \sqrt{28} = 8\sqrt{2}$

and

$\angle A = ?,\ \angle B = ?,\ \angle C = 90°$

$\tan(\angle A) = \frac{a}{b}$

$\Rightarrow \tan(\angle A) = \frac{a}{b} = \frac{8}{8} = 1$

$\angle A = \tan^{-1}(1)$

$\angle A = 45°$

$\angle A = 45°,\ \angle C = 90°,\ \angle B = ?$

$\angle A + \angle B + \angle C = 180°$

$\Rightarrow \angle B = 180° - \angle A - \angle C$

$= 180° - 45° - 90° = 45°$

$\Rightarrow \angle B = 45°$

hence

$\angle A = 45°$ a=1

$\angle B = 45°$ b = 8

$\angle C = 90°$ c = 85

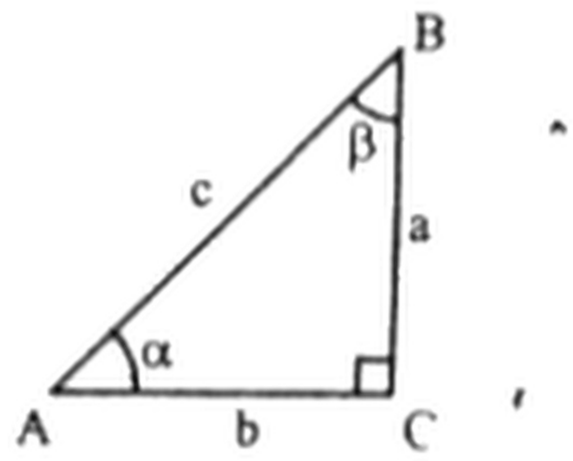
Solve the right triangle ABC, in which $\gamma = 90°$

2. α = 37° 20', a=243 **3. α = 62° 40', b = 796**

4. a= 3.28, b= 5.74 **5. b = 68.4, c = 96.2**

6. .a=5429, c=6294 **7. β = 50° 10', c = 0.832**

2. a=37° 20', γ =90°, a=243



Solution

α = 37° 20', γ = 90°, β = ?

we know that

$\alpha + \beta + \gamma = 180°$

$\Rightarrow \beta = 180° - \gamma - \alpha$

$= 180° - 90° - 37° 20' = 52° 40'$

$\Rightarrow \beta = 52° 40'$

and

a = 243, b = ?, c= ?

$$\sin(37^\circ 20') = \frac{a}{c}$$

$$\Rightarrow c = \frac{a}{\sin(37^\circ 20')} = \frac{243}{0.666} = 400.9$$

$$\tan(37^\circ 20') = \frac{a}{b}$$

$$\Rightarrow b = \frac{a}{\tan(37^\circ 20')} = \frac{243}{0.7627} = 318.50$$

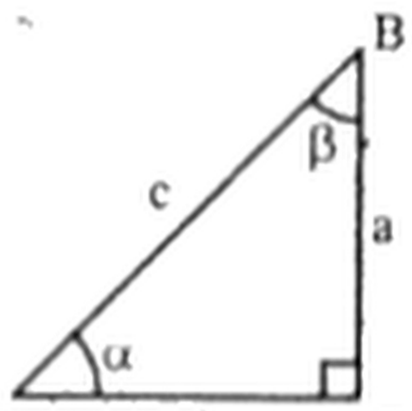
Hence

$a = 243$ $\quad \alpha = 37^\circ 20'$

$b = 318.60$ $\quad \beta = 52^\circ 40'$

$c = 400.9$ $\quad \gamma = 90^\circ$

3. $\alpha = 62^\circ 40'$, $\beta = 796$, $\gamma = 90^\circ$



Solution

we know that

$\alpha + \beta + \gamma = 180^\circ$

$\beta = 180^\circ - 62^\circ 40 - 90^\circ = 27^\circ 20^\circ$

$\Rightarrow \beta = 27^\circ 20'$.

and

$$\cos\alpha = \frac{b}{c}$$

$$\Rightarrow c = \frac{b}{\cos\alpha} = \frac{796}{0.459} = 1733.58$$

$\Rightarrow c = 1733.58$

$$\tan \alpha = \frac{a}{b}$$

$\Rightarrow a = b \tan \alpha$

$= 796 \tan (62° 40°)$

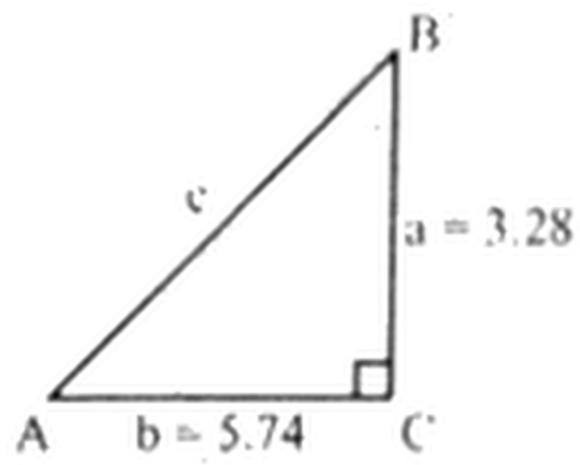
$= 796 (1.9347) = 1540.02$

$\Rightarrow a = 1540.02$

Hence

$\alpha = 62° 40$	a = 1540.02
$\beta = 27° 20'$	b = 796
$\gamma = 90°$	c = 1733.58

4. a = 3.28, b= 5.74, γ = 90°



Solution

a = 3.28, b = 5.74. c =?

we know that

$c^2 = a^2 + b^2$

$= (3.282)^2 + (5.74)^2$

= 10.7584 + 32.9476

= 43.706

$\Rightarrow c = \sqrt{43.706} = 6.611$

and

$$\tan(\alpha) = \frac{a}{b} = \frac{3.28}{5.74}$$

$\alpha = \tan^{-1}(0.5714)$

$\Rightarrow \alpha = 29.74 = 29° 44'$

$\alpha + \beta + \gamma = 180°$

$\beta = 180° - \alpha - \gamma$

$= 180° - 29° 44' - 90° = 60° 16'$

$\Rightarrow \beta = 60° 16'$

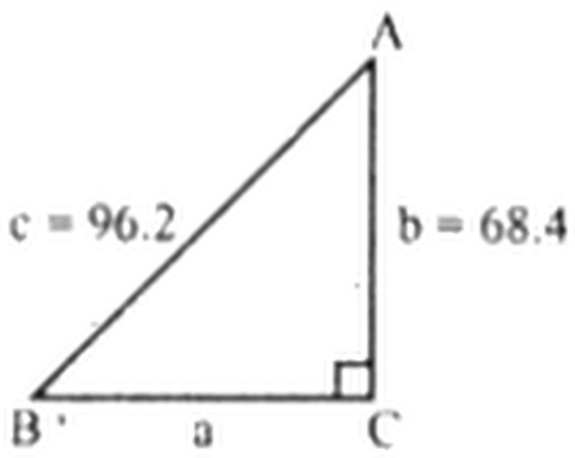
Hence

a = 3.28 $\alpha = 29° 44'$

b = 5.74 $\beta = 60° 16'$

c = 6.611 $\gamma = 90°$

5. b = 68.4, c = 96.2, $\gamma = 90°$



Solution

a = 68.4, b = 96.2, a = ?

We know that by Pythagoras theorem.

$c^2 = a^2 + b^2$

$\Rightarrow a^2 = c^2 - b^2$

$= (96.2)^2 + (68.4)^2$

$= 9254.44 - 4678.56$

$a^2 = 4575.88$

$\Rightarrow a = 67.64$

and

$\sin \alpha = \frac{b}{c}$

$= \frac{68.4}{96.2} = 0.71$

$\alpha = \sin^{-1}(0.71) = 45.317$

$\Rightarrow \alpha = 45° 19$

we know that

$\alpha + \beta + \gamma = 180°$

$\beta = 180° - \alpha - \gamma$

$= 180° - 45° 19 - 90° = 44° 40'$

$\Rightarrow \beta = 44° 40'$

Hence

$\alpha = 45° 19'$	a = 67.64
$\beta = 44° 41'$	b = 68.4
$\gamma = 90°$	c = 962

6. a= 5429, c=6294, γ = 90°

Solution

a = 5429, c = 6294, a = ?

We know that by Pythagoras theorem,

$c^2 = a^2 + b^2$

$\Rightarrow b^2 = c^2 - b^2$

$= (6294)^2 + (5429)^2$

= 39614436 - 29474041

= 10140395

$\Rightarrow$ b = $\sqrt{10140395}$ = 3184.40

and

$\sin\alpha = \frac{a}{c} = \frac{5429}{6294}$

$\sin\alpha$ = 0.86256

$\alpha = \sin^{-1}(0.86256)$

= 59.60 = 59° 36'

$\Rightarrow \alpha$ = 59° 36'

we know that

$\alpha + \beta + \gamma = 180°$

$\beta = 180° - \gamma - \alpha$

= 180°-90° - 59° 36' = 30° 24'

$\Rightarrow \beta$ = 30° 24'

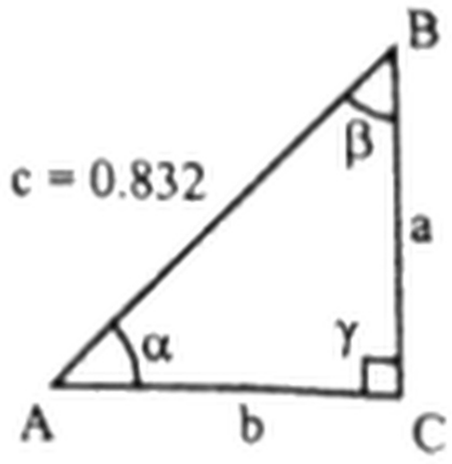
Hence

a= 5429 α = 59° 36'

b = 3184.40 β = 30° 24'

c = 6294 γ =90°

7. β =50° 10', c=0.832, γ =90°



Solution

β = 50° 10', γ = 90°, β = ?

we know that

$\alpha + \beta + \gamma = 180°$

$\Rightarrow \alpha = 180° - \gamma - \beta$

= 180°-90°-50° 10' = 39° 50'

$\Rightarrow \alpha$ = 39° 50'

a=?, b=?, c=0.832

$\sin \beta = \frac{b}{c}$

$\Rightarrow$b = c sin β

= 0.832 sin (50° 10')

= 0.6389

$\Rightarrow$b = 0.6389

$\cos \beta = \frac{a}{c}$

$\Rightarrow$a = c cos β

= 0.832 cos (50° 10')

= 0.533

$\Rightarrow$a = 0.533

hence

a = 0.533 α = 39° 50'

$b = 0.6389 \qquad \beta = 50° 10'$

$C = 0.832 \qquad \gamma = 90°$