Exercise 12.1

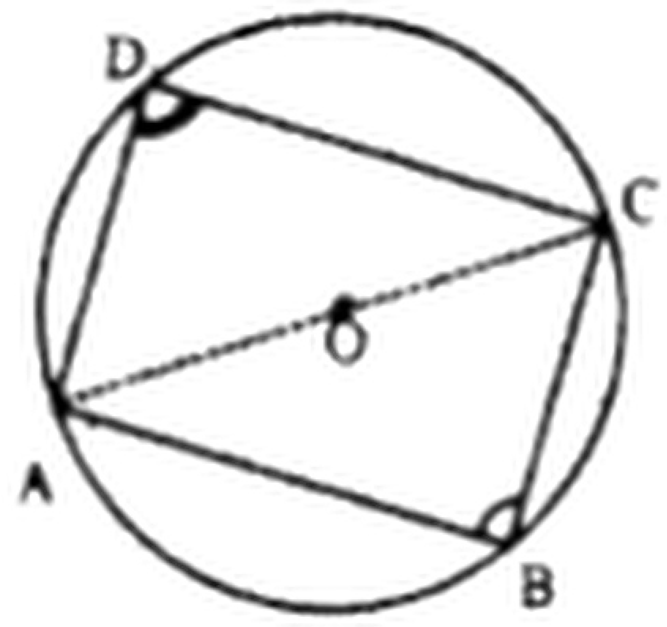
1. Prove that in a given cyclic quadrilateral, sum of opposite angles is two right angles and conversely.

Solution:

Given:

A quadrilateral ABCD.

In scribed in a circle whose centre is O.



To Prove:

$\angle A + \angle C = 2\angle rts.$

And $\angle B + \angle D = 2\angle rts.$

Construction:

Join A with O and O with C

Proof:

Arc ABC subtends $\angle AOC$ at the centre O and.

$\angle$ ADC at a point D on the remaining part of the circumference.

$$m\angle ADC = \frac{1}{2}\angle AOC \qquad ....(i)$$

Similarly, arc ADC subtends reflex ∠AOC at the centre and ∠ABC on the circumferences.

$$m\angle ABC = \frac{1}{2}\angle AOC \qquad \ldots\ldots(ii)$$

By Adding (i) & (ii)

$$m\angle ADC + m\angle ABC = \frac{1}{2}[m\angle AOC + m\angle AOC]$$

$$m\angle D + m\angle B = \frac{1}{2}[4\angle rts]$$

$m\angle D + m\angle B = 2rt\ \angle S.$ Proved.

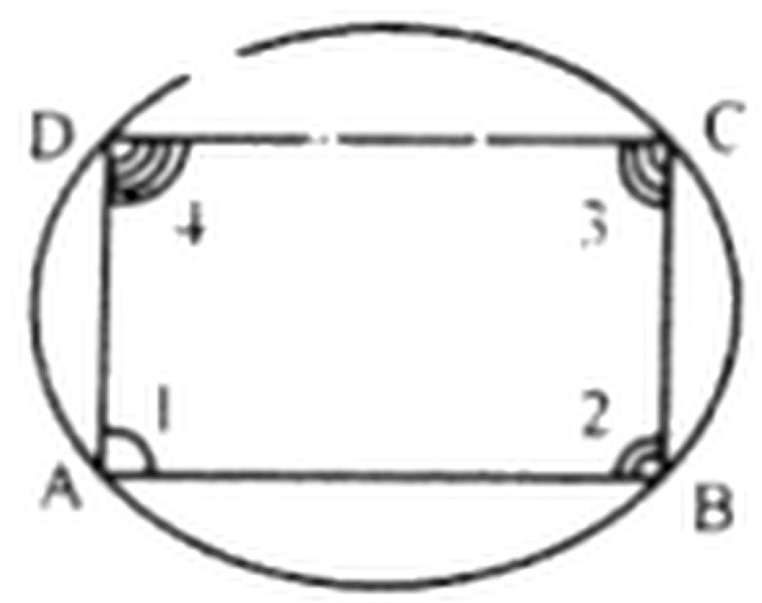
Similarly, by Joining B with O and O with D it can be proved that

$\angle A + \angle C = 2\angle rts.$

2. Show that parallelogram inscribed in a circle will be a rectangle.

Solution:

Given:



ABCD is a parallelogram inscribed in a circle with centre "O".

To Prove:

ABCD is a rectangle.

Proof

$m\angle 1 + m\angle 3 = 180°$ __________(i)

[cyclic quadrilateral]

But $m\angle 1 + m\angle 3 = 180°$ __________(ii)

[opp. ∠sof a parallelogram)

From (i) & (ii) we get

$$2m\angle 1 = 180°$$

$\Rightarrow$ $m\angle 1 = 90°$

$\therefore m\angle 1 = m\angle 3 = 90°$.

By this

$m\angle 1 = m\angle 2 = m\angle 3 = m\angle 4 = 90°$.

Hence, the parallelogram ABCD is a rectangle.

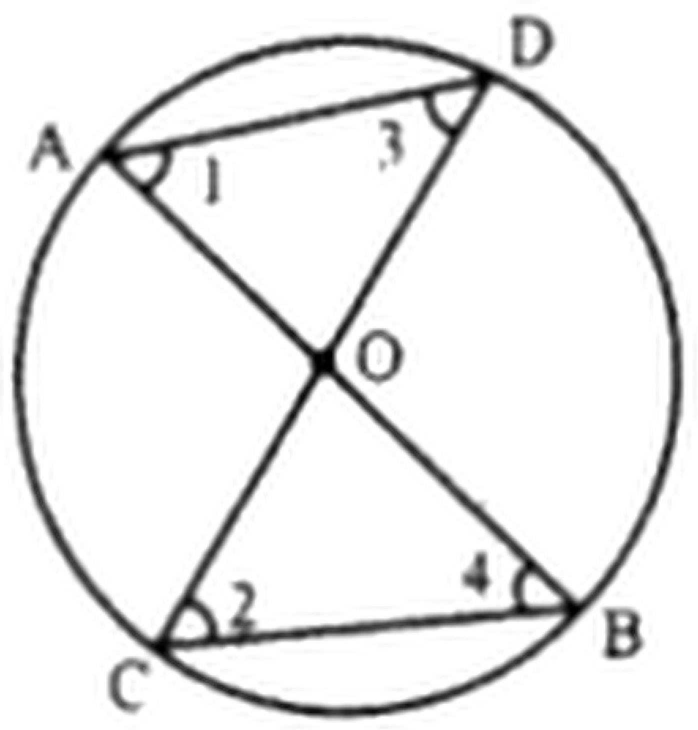
3. AOB and COD are two intersecting chords of a circle. Show that $\triangle$s AOD and BOC are equiangular.

Solution:

Given:

Two chords AOB & COD

Intersecting each other at O.



To Prove:

$\triangle$AOD and $\triangle$COD are equiangular.

Proof:

$\triangle$AOB and $\triangle$COB

∠1 = ∠2.

∠3 = ∠4

[angles in the same segment of a circle].

∴∠AOB = ∠COB.

(Vertical angles).

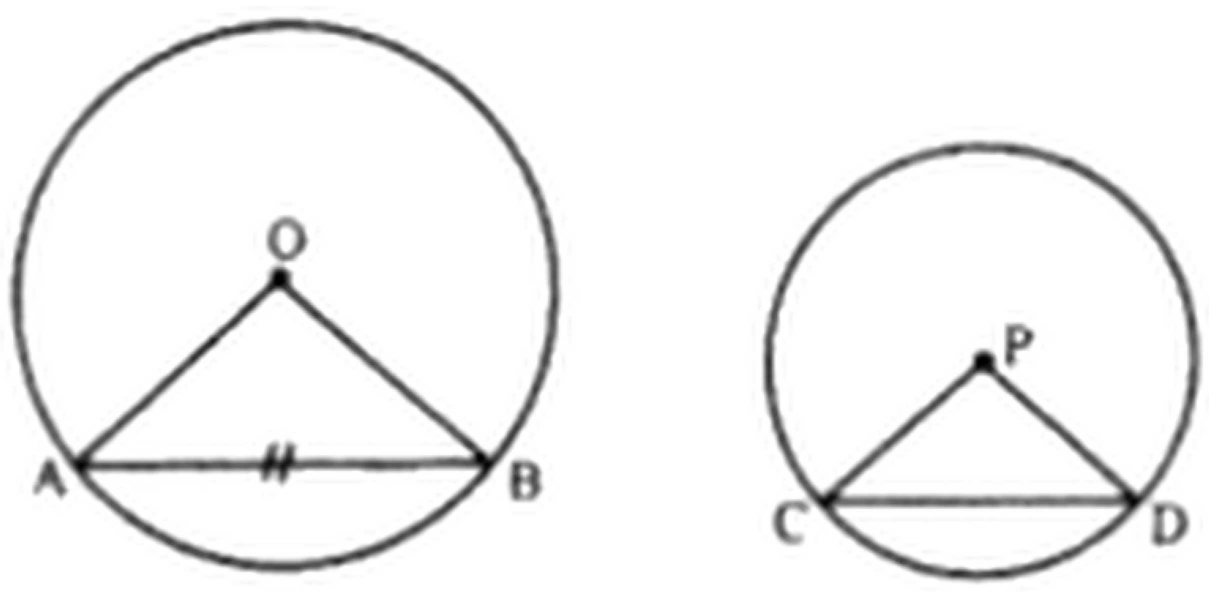
△AOD and △COD are equiangular triangles.

4. $\overline{AD}$, and $\overline{BC}$ are two parallel chords of a circle prove that arc $\overline{AB}$ ≅ arc $\overline{CD}$ and arc $\overline{AC}$ ≅ arc $\overline{BD}$.

Solution:

Given:

Two circles, C (O, r) and C (P, r) chord AB = chord CD.



To Prove:

$\overset{\frown}{AB} = \overset{\frown}{CD}$

Construction:

Join OA, OB and PC, PD.

Proof:

In △AOB & △CPD

$OA \cong PC$

and $OB \cong PD$ [radii of equal circles]

$AB = CD$ [given]

$\therefore \triangle AOB \cong \triangle CPD$ S.S.S.

$\therefore \angle AOB \cong \angle CPD$

There are the angles subtended by the AB and CD at the centre of equal circle

$\overset{\frown}{AB} = \overset{\frown}{CD}$