Exercise 10.4

1. Express the following products as sums or differences:

i. $2\sin 3\theta \cos\theta$ **ii.** $2\cos 5\theta \sin 3\theta$

iii. $\sin 5\theta \cos 2\theta$ **iv.** $2\sin 7\theta \sin 2\theta$

v. $\cos(x+y)\sin(x-y)$ **vi.** $\cos(2x+30^0)\cos(2x-30^0)$

vii. $\sin 12^0 \sin 46^0$ **viii.** $\sin(x+45^0)\sin(x-45^0)$

i. $2\sin 3\theta \cos\theta$

Solution:

$$2\sin 3\theta \cos\theta = \sin(3\theta+\theta)+\sin(3\theta-\theta)$$

$$\Rightarrow \qquad = \sin 4\theta + \sin 2\theta$$

Hence

$$2\sin 3\theta \cos\theta = \sin 4\theta + \sin 2\theta$$

ii. $2\cos 5\theta . \sin 3\theta$

Solution:

$$2\cos 5\theta . \sin 3\theta = \sin(5\theta+3\theta)-\sin(5\theta-3\theta)$$

$$= \sin 8\theta - \sin 2\theta$$

$$= \frac{1}{2}$$

Hence,

$$2\cos 5\theta.\sin 3\theta = Sin8\theta - Sin2\theta$$

iii. $2\cos 5\theta.\sin 2\theta$

Solution:

$$\sin 5\theta.\cos 2\theta = \frac{1}{2}\left[2\sin 5\theta.\cos 2\theta\right]$$

$$= \frac{1}{2}\left[\sin\left(5\theta + 2\theta\right) - \sin\left(5\theta - 2\theta\right)\right]$$

$$= \frac{1}{2}\left[\sin 7\theta - \sin 3\theta\right]$$

Hence,

$$\sin 5\theta.\cos 2\theta = \frac{1}{2}\left[\sin 7\theta - \sin 3\theta\right]$$

iv. $2\sin 7\theta.\sin^2 \theta$

Solution:

Question not prearranged.

$$2\sin 7\theta \sin 2\theta = -\left[\cos\left(7\theta + 2\theta\right) - \cos\left(7\theta - 2\theta\right)\right]$$

$$= -\left[\cos\left(9\theta\right) - \cos\left(5\theta\right)\right]$$

$$= \cos 5\theta - \cos 9\theta$$

Hence,

$$2\sin 7\theta.\sin 2\theta = \cos 5\theta - \cos 9\theta$$

v. $\cos\left(x+y\right).\sin\left(x-y\right)$

Solution:

$$\cos(x+y)\sin(x-y)=\frac{1}{2}\left[\sin(x+y+x-y)-\sin(x+y-x+y)\right]$$

$$=\frac{1}{2}\left[\sin(2x)-\sin(2y)\right]$$

Hence

$$\cos(x+y)\sin(x-y)=\frac{1}{2}\left[\sin 2x-\sin 2y\right]$$

vi. $\cos(2x+30^0)\cos(2x-30^0)$

Solution:

$$\cos(2x+30^0).\cos(2x-30^0)=\frac{1}{2}\left[2\cos(2x+30^0).(2x-30^0)\right]$$

$$=\frac{1}{2}\left[\cos(2x+30^0+2x-30^0)+\cos(2x+30^0+2x-30^0)\right]$$

$$=\frac{1}{2}\left[\cos(4x)+\cos 60^0\right]$$

$$=\frac{1}{2}\left[\cos 4x+\cos 60^0\right]$$

Hence,

$$\cos(2x+30^0).\cos(2x-30^0)=\frac{1}{2}\left[\cos 4x+\cos 60^0\right]$$

vii. $\sin 12^0\sin 46^0$

Solution:

$$\sin 12^0.\sin 46=\frac{-1}{2}\left[-2\sin 12^0.\sin 46^0\right]$$

$$= \frac{-1}{2}\left[\cos\left(12^0 + 46^0\right) - \cos\left(12^0 - 46^0\right)\right]$$

$$= \frac{-1}{2}\left[\cos 58^0 - \cos\left(-34^0\right)\right]$$

$$= \frac{-1}{2}\left[\cos 58^0 - \cos 34^0\right]$$

Hence $\sin 12^0 . \sin 46^0 = \frac{-1}{2}\left[\cos 58^0 - \cos 34^0\right]$

viii. $\sin\left(x + 45^0\right) . \sin\left(x - 45^0\right)$

Solution:

$$\sin\left(x + 45^0\right) . \sin\left(x - 45^0\right) = \frac{-1}{2}\left[-2\sin\left(x + 45^0\right)\sin\left(x - 45^0\right)\right]$$

$$= \frac{-1}{2}\left[\cos\left(x + 45^0 + x - 45^0\right) - \cos\left(x + 45^0 - x + 45^0\right)\right]$$

$$= \frac{-1}{2}\left[\cos\left(2x\right) - \cos\left(90^0\right)\right]$$

$$= \frac{-1}{2}\cos 2x$$

Hence $\sin\left(x + 45^0\right) . \sin\left(x - 45^0\right) = \frac{-1}{2}\cos 2x$

2. Express the following sums or differences as products.

i. $\sin 5\theta + \sin 3\theta$ **ii.** $\sin 8\theta - \sin 4\theta$

iii. $\cos 6\theta + \cos 3\theta$ **iv.** $\cos 7\theta - \cos\theta$

v. $\cos 12^0 + \cos 48^0$ **vi.** $\sin\left(x + 30^0\right) + \sin\left(x - 30^0\right)$

i. $\sin 5\theta + \sin 3\theta$

Solution:

$$\sin 5\theta + \sin 3\theta = 2\sin\frac{5\theta + 3\theta}{2}\cos\frac{5\theta - 3\theta}{2}$$

$$= 2\sin\frac{80}{2}\cos\theta$$

$$= 2\sin 40\cos\theta$$

Hence, $\sin 5\theta + \sin 3\theta \;= 2\sin 40\cos\theta$

ii. $\sin 8\theta - \sin 4\theta$

Solution:

$$\sin 8\theta - \sin 4\theta = 2\cos\frac{8\theta + 4\theta}{2}\sin\frac{8\theta - 4\theta}{2}$$

$$= 2\cos\frac{12\theta}{2}.\sin\frac{4\theta}{2}$$

$$= 2\cos 6\theta.\sin 2\theta$$

Hence; $\sin 8\theta - \sin 4\theta = 2\cos 6\theta.\sin 2\theta$

iii. $\cos 6\theta + \cos 3\theta$

Solution:

$$\cos 6\theta + \cos 3\theta = 2\cos\frac{6\theta + 3\theta}{2}.\cos\frac{6\theta - 3\theta}{2}$$

$$= 2\cos\frac{9\theta}{2}\cos\frac{3\theta}{2}$$

Hence; $\cos 6\theta + \cos 3\theta = 2\cos\frac{9\theta}{2}\cos\frac{3\theta}{2}$

iv. $\cos 7\theta - \cos\theta$

Solution:

$$\cos 7\theta - \cos\theta = -2\sin\frac{7\theta+\theta}{2}.\sin\frac{7\theta-\theta}{2}$$

$$= -2\sin\frac{8\theta}{2}.\sin\frac{6\theta}{2}$$

$$= -2\sin 4\theta \sin 3\theta$$

Hence; $\cos 7\theta - \cos\theta = -2\sin 4\theta.\sin 3\theta$

v. $\cos 12^0 + \cos 48^0$

Solution:

$$\cos 12^0 + \cos 48^0 = -2\cos\frac{12^0+48^0}{2}\cos\frac{12^0-48^0}{2}$$

$$= -2\cos\frac{60^0}{2}\cos\left(\frac{-36^0}{2}\right)$$

$$= -2\cos 30^0\left(+\cos 18^0\right)$$

$$= 2\cos 30^0.\cos 18^0$$

Hence; $\cos 12^0 + \cos 48^0 = 2\cos 30^0 \cos 18^0$

vi. $\sin\left(x+30^0\right)+\sin\left(x-30^0\right)$

Solution:

$$\sin\left(x+30^0\right)+\sin\left(x-30^0\right)$$

$$= 2\sin\left(x+30^0+x-30^0\right)\cos\left(x+30^0 \Rightarrow -x+30^0\right)$$

$$= 2\sin 2x.\cos 60^{0}$$

Hence;

$$\sin\left(x + 30^{0}\right) + \sin\left(x - 30^{0}\right) = 2\sin 2x.\cos 60^{0}$$

3. Prove that following identities:

i. $\dfrac{\sin 3x - \sin x}{\cos x - \cos 3x} = \cot 2x$

ii. $\dfrac{\sin 8x + \sin 2x}{\cos 8x + \cos 2x} = \tan 5x$

iii. $\dfrac{\sin\alpha - \sin\beta}{\cos 8\alpha + \cos\beta} = \tan\dfrac{\alpha - \beta}{2}\cot\dfrac{\alpha + \beta}{2}$

i. $\dfrac{\sin 3x - \sin x}{\cos x - \cos 3x} = \cot 2x$

Solution:

L.H.S $= \dfrac{\sin 3x - \sin x}{\cos x - \cos 3x}$

$$= \frac{2\cos\left(\frac{3x + x}{2}\right)\sin\left(\frac{3x - x}{2}\right)}{-2\sin\left(\frac{x + 3x}{2}\right)\sin\left(\frac{x - 3x}{2}\right)}$$

$$= \frac{2\cos\left(\frac{4x}{2}\right)\sin\left(\frac{2x}{2}\right)}{-2\sin\left(\frac{4x}{2}\right)\sin\left(\frac{-2x}{2}\right)}$$

$$= \frac{2\cos 2x.\sin x}{2\sin 2x.\sin x}$$

$$= \frac{\cos 2x}{\sin 2x}$$

$$= \cot 2x$$

$= \text{R.H.S}$

Hence proved

$$\frac{\sin 3x - \sin x}{\cos x - \cos 3x} = \cot 2x$$

ii. $$\frac{\sin 8x + \sin 2x}{\cos 8x + \cos 2x} = \tan 5x$$

Solution:

$$\text{L.H.S} = \frac{\sin 8x + \sin 2x}{\cos 8x + \cos 2x}$$

$$= \frac{2\sin\left(\frac{8x+2x}{2}\right).\cos\left(\frac{8x-2x}{2}\right)}{2\cos\left(\frac{8x+2x}{2}\right).\cos\left(\frac{8x-2x}{2}\right)}$$

$$= \frac{2\sin\left(\frac{10x}{2}\right).\cos\left(\frac{6x}{2}\right)}{2\cos\left(\frac{10x}{2}\right).\cos\left(\frac{6x}{2}\right)}$$

$$= \frac{\sin 5x}{\cos 5x}$$

$$= \tan 5x$$

$= \text{R.H.S}$

Hence Proved

$$\frac{\sin 8x + \sin 2x}{\cos 8x + \cos 2x} = \tan 5x$$

iii. $\frac{\sin\alpha - \sin\beta}{\cos\alpha + \cos\beta} = \tan x \frac{\alpha - \beta}{2} \cot\frac{\alpha + \beta}{2}$

Solution:

$$\text{L.H.S} = \frac{\sin\alpha - \sin\beta}{\cos 8\alpha + \cos 2\beta}$$

$$= \frac{2\cos\frac{\alpha+\beta}{2}.\sin\frac{\alpha-\beta}{2}}{2\sin\frac{\alpha+\beta}{2}.\cos\frac{\alpha-\beta}{2}}$$

$$= \frac{\cos\frac{\alpha+\beta}{2}.\sin\frac{\alpha-\beta}{2}}{\sin\frac{\alpha+\beta}{2}.\cos\frac{\alpha-\beta}{2}}$$

$$= \cot\frac{\alpha+\beta}{2}.\tan\frac{\alpha-\beta}{2}$$

= R.H.S

Hence proved

$$\frac{\sin\alpha - \sin\beta}{\cos 8\alpha + \cos 2\beta} = \tan\frac{\alpha-\beta}{2}\cot\frac{\alpha+\beta}{2}$$

4. Prove that

i. $\cos 20^0 \cos 100^0 \cos 140^0 = 0$

ii. $\sin\left(\frac{\pi}{4} - \theta\right)\sin\left(\frac{\pi}{4} + \theta\right) = \frac{1}{2}\cos 2\theta$

iii. $\frac{\sin\theta + \sin 3\theta + \sin 5\theta + \sin 7\theta}{\cos\theta + \cos 3\theta + \cos 5\theta + \cos 7\theta} = \tan 4\theta$

i. $\cos 20^0 \cos 100^0 \cos 140^0 = 0$

Solution:

$$\text{L.H.S} = \cos 20^0 \cos 100^0 \cos 140^0$$

$$= 2\cos\left(\frac{20+100^0}{2}\right)\cos\left(\frac{20-100^0}{2}\right)+\cos 140^0$$

$$= 2\cos 60^0 \cos(-40^0)+\cos(180^0-40^0)$$

$$= 2\left[\frac{1}{2}\cos(40^0)\right]-\cos 40$$

$$= \cos 40 - \cos 40$$

$$= 0$$

$$= \text{R.H.S}$$

Hence proved

$$\cos 20^0 . \cos 100^0 . \cos 140^0 = 0$$

ii. $\sin\left(\frac{\pi}{4}-\theta\right)\sin\left(\frac{\pi}{4}+\theta\right)=\frac{1}{2}\cos 2\theta$

Solution:

$$\text{L.H.S} = \sin\left(\frac{\pi}{4}-\theta\right)\sin\left(\frac{\pi}{4}+\theta\right)$$

$$= \left(\sin\frac{\pi}{4}.\cos\theta-\cos\frac{\pi}{4}\sin\theta\right)\left(\sin\frac{\pi}{4}\cos\theta+\cos\frac{\pi}{4}.\sin\theta\right)$$

$$= \left(\frac{1}{\sqrt{2}}.\cos\theta-\frac{1}{\sqrt{2}}\sin\theta\right)\left(\frac{1}{\sqrt{2}}\cos\theta+\frac{1}{\sqrt{2}}.\sin\theta\right)$$

$$= \frac{1}{\sqrt{2}}\left[\cos\theta-\sin\theta\right]\frac{1}{\sqrt{2}}\left[\cos\theta+\sin\theta\right]$$

$$= \frac{1}{2}\left(\cos^2\theta - \sin^2\theta\right)$$

$$= \frac{1}{2}\cos 2\theta$$

= R.H.S

Hence; $\sin\left(\frac{\pi}{4} - \theta\right)\sin\left(\frac{\pi}{4} + \theta\right) = \frac{1}{2}\cos 2\theta$

iii. $\frac{\sin\theta + \sin 3\theta + \sin 5\theta + \sin 7\theta}{\cos\theta + \cos 3\theta + \cos 5\theta + \cos 7\theta} = \tan 4\theta$

Solution:

$$\text{L.H.S} = \frac{\sin\theta + \sin 3\theta + \sin 5\theta + \sin 7\theta}{\cos\theta + \cos 3\theta + \cos 5\theta + \cos 7\theta}$$

$$= \frac{(\sin 7\theta + \sin\theta) + (\sin 5\theta + \sin 3\theta)}{(\cos 7\theta + \cos\theta) + (\cos 5\theta + \cos 3\theta)}$$

$$= \frac{2\sin\left(\frac{7\theta+\theta}{2}\right).\cos\left(\frac{7\theta-\theta}{2}\right) + 2\sin\left(\frac{5\theta+3\theta}{2}\right)\cos\left(\frac{5\theta-3\theta}{2}\right)}{2\cos\left(\frac{7\theta+\theta}{2}\right).\cos\left(\frac{7\theta-\theta}{2}\right) + 2\sin\left(\frac{7\theta+\theta}{2}\right)\cos\left(\frac{5\theta-3\theta}{2}\right)}$$

$$= \frac{2\sin 4\theta.\cos 3\theta + 2\sin 4\theta.\cos\theta}{2\cos 4\theta.\cos 3\theta + 2\cos 4\theta.\cos\theta}$$

$$= \frac{\sin 4\theta}{\cos 4\theta}\left[\frac{2\cos 3\theta + 2\cos\theta}{2\cos 3\theta + 2\cos\theta}\right]$$

$$= \tan 4\theta$$

= R.H.S

Hence proved $\frac{\sin\theta + \sin 3\theta + \sin 5\theta + \sin 7\theta}{\cos\theta + \cos 3\theta + \cos 5\theta + \cos 7\theta} = \tan 4\theta$

5. Prove that:

i. $\cos 20^0 \cos 40^0 \cos 60^0 \cos 80^0 = \frac{1}{16}$

ii. $\sin\frac{\pi}{9}\sin\frac{2\pi}{9}\sin\frac{2\pi}{3}\sin\frac{4\pi}{9} = \frac{3}{16}$

iii. $\sin 10^0 \sin 30^0 \sin 50^0 \sin 70^0 = \frac{1}{16}$

i. $\cos 20^0 \cos 40^0 \cos 60^0 \cos 80^0 = \frac{1}{16}$

Solution:

$$\text{L.H.S} = \cos 20^0 \cos 40^0 \cos 60^0 \cos 80^0$$

$$= \left(\cos 80^0 \cos 20^0\right)\left(\cos 60^0.\cos 40^0\right)$$

$$= \frac{1}{2}\left[2\cos 80^0 \cos 20^0\right]\left[\cos 60^0.\cos 40^0\right]$$

$$= \frac{1}{4}\left[\cos\left(80^0 + 20^0\right) + \cos\left(80^0 - 20^0\right)\right]\left[\frac{1}{2}.\cos 40^0\right]$$

$$= \frac{1}{4}\left[\cos\left(100^0\right) + \cos\left(60^0\right)\right]\left[\frac{1}{2}\cos 40^0\right]$$

$$= \frac{1}{4}\left[\cos 100^0.\cos 40^0 + \frac{1}{2}\cos 40^0\right]$$

$$= \frac{1}{4}\left[\frac{1}{2}\left(\cos 100^0.\cos 40^0\right) + \frac{1}{2}\cos 40^0\right]$$

$$= \frac{1}{8}\left[\left(2\cos 100^0.\cos 40^0\right) + \cos 40^0\right]$$

$$= \frac{1}{8}\left[\cos\left(100^0 + 40^0\right) + \left(\cos 100^0 - 40^0\right) + \cos 40^0\right]$$

$$= \frac{1}{8}\left[cos140^0 + \cos 60^0 + \cos 40^0\right]$$

$$= \frac{1}{8}\left[cos140^0 + \frac{1}{2} + \cos 40^0\right]$$

$$= \frac{1}{8}\left[\frac{1}{2} + cos140^0 + \cos 40^0\right]$$

$$= \frac{1}{16} + \frac{1}{8}\left[cos140^0 + \cos 40^0\right]$$

$$= \frac{1}{16} + \frac{1}{8}\left[2.cos\frac{140^0 + 40^0}{2}.\cos\frac{140^0 - 40^0}{2}\right]$$

$$= \frac{1}{16} + \frac{1}{8}\left[2.cos90^0.\cos 50^0\right]$$

$$= \frac{1}{16} + \frac{1}{8}\left[2.(0).\cos 50^0\right]$$

$$= \frac{1}{16} + \frac{1}{8}(0)$$

$$= \frac{1}{16} + 0$$

$$= \frac{1}{16}$$

= R.H.S

Hence proved

$$\cos 20^0.\cos 40^0.\cos 60^0.\cos 80^0 = \frac{1}{16}$$

ii. $\sin\frac{\pi}{9}\sin\frac{2\pi}{9}\sin\frac{2\pi}{3}\sin\frac{4\pi}{9} = \frac{3}{16}$

Solution:

$$\text{L.H.S} = \sin\frac{\pi}{9}\sin\frac{2\pi}{9}\sin\frac{2\pi}{3}\sin\frac{4\pi}{9}$$

$$= \left(\sin\frac{4\pi}{9}\sin\frac{\pi}{9}\right).\sin\frac{2\pi}{9}\left(\sqrt{3/2}\right)$$

$$= \frac{-\sqrt{3}}{4}\left(-2\sin\frac{4\pi}{9}\sin\frac{\pi}{9}\right).\sin\frac{2\pi}{9}$$

$$= \frac{-\sqrt{3}}{4}\left(\cos\left(\frac{4\pi}{9}+\frac{\pi}{9}\right)-\cos\left(\frac{4\pi}{9}-\frac{\pi}{9}\right)\right)\sin\frac{2\pi}{9}$$

$$= \frac{-\sqrt{3}}{4}\left(\cos\frac{5\pi}{9}-\cos\frac{3\pi}{9}\right)\sin\frac{2\pi}{9}$$

$$= \frac{-\sqrt{3}}{4}\left(\cos\frac{5\pi}{9}-\frac{1}{2}\right).\sin\frac{2\pi}{9}$$

$$= \frac{-\sqrt{3}}{4}\left(\cos\frac{5\pi}{9}.\sin\frac{2\pi}{9}\right)+\frac{\sqrt{3}}{8}.\sin\frac{2\pi}{9}$$

$$= \frac{-\sqrt{3}}{8}\sin\left(\cos\frac{5\pi}{9}+\sin\frac{2\pi}{9}\right)-\sin\left(\cos\frac{5\pi}{9}-\sin\frac{2\pi}{9}\right)+\frac{\sqrt{3}}{8}.\sin\frac{2\pi}{9}$$

$$= \frac{-\sqrt{3}}{8}\left(\sin\frac{7\pi}{9}.\sin\frac{3\pi}{9}\right)+\frac{\sqrt{3}}{8}.\sin\frac{2\pi}{9}$$

$$= \frac{-\sqrt{3}}{8}\sin\frac{7\pi}{9}+\frac{3}{16}+\frac{\sqrt{3}}{8}\sin\frac{2\pi}{9}$$

$$= \frac{-\sqrt{3}}{8}\left[\sin\frac{7\pi}{9}-\sin\frac{7\pi}{9}\right]+\frac{3}{16}$$

$$= \frac{-\sqrt{3}}{8}\left[2.\cos\frac{7\pi/9+2\pi/9}{2}.\sin\frac{7\pi/9-2\pi/9}{2}\right]+\frac{3}{16}$$

$$= \frac{-\sqrt{3}}{8}\left[2\cos\frac{\pi}{9}.\sin\frac{5\pi}{18}\right]+\frac{3}{16}$$

$$= \frac{-\sqrt{3}}{8}\left[2(0).\sin\frac{5\pi}{18}\right] + \frac{3}{16}$$

$$= 0 + \frac{3}{16} = \frac{3}{16}$$

= R.H.S

Hence proved

$$\sin\frac{\pi}{9}\sin\frac{2\pi}{9}\sin\frac{2\pi}{3}\sin\frac{4\pi}{9} = \frac{3}{16}$$

iii. $\sin 10^0 \sin 30^0 \sin 50^0 \sin 70^0 = \frac{1}{16}$

Solution:

L.H.S $= \sin 10^0 \sin 30^0 \sin 50^0 \sin 70^0$

$$= \left(\sin 70^0.\sin 10^0\right)\frac{1}{2}.\sin 50^0$$

$$= \frac{-1}{4}\left[-2\sin 70^0 \sin 10^0\right].\sin 50^0$$

$$= \frac{-1}{4}\left[\cos\left(70^0 + 10^0\right) - \cos\left(70^0 - 10^0\right)\right]\sin 50^0$$

$$= \frac{-1}{4}\left[\cos 80^0 - \cos 60^0\right]\sin 50^0$$

$$= \frac{-1}{4}\left[\cos 80^0 - \frac{1}{2}\right]\sin 50^0$$

$$= \frac{-1}{4}\cos 80^0.\sin 50^0 + \frac{1}{8}.\sin 50^0$$

$$= \frac{-1}{8}\left[2\cos 80^0 - \sin 50^0\right] + \frac{1}{8}\sin 50^0$$

$$= \frac{-1}{8}\left[\sin\left(80^0+50^0\right)-\sin\left(80^0-50^0\right)\right]+\frac{1}{8}\sin 50^0$$

$$= \frac{-1}{8}\left(\sin 130^0-\sin 30^0\right)+\frac{1}{8}\sin 50^0$$

$$= \frac{-1}{8}\left(\sin 130^0-\frac{1}{2}\right)+\frac{1}{8}\sin 50^0$$

$$= \frac{-1}{8}\sin 130^0+\frac{1}{16}+\frac{1}{8}\sin 50^0$$

$$= \frac{1}{8}\left[\sin 50^0-\sin 130^0\right]+\frac{1}{16}$$

$$= \frac{1}{8}\left[2\cos\frac{50^0+130^0}{2}.\sin\frac{50^0-130^0}{2}\right]+\frac{1}{16}$$

$$= \frac{1}{8}\left[2\cos 90^0.\sin(-40^0)\right]+\frac{1}{16}$$

$$= \frac{-1}{8}\left[2(0)\sin(40^0)\right]+\frac{1}{16}$$

$$= 0+\frac{1}{16}$$

$$= \frac{1}{16}$$

= R.H.S

Hence proved

$$\sin 10^0 \sin 30^0 \sin 50^0 \sin 70^0 = \frac{1}{16}$$