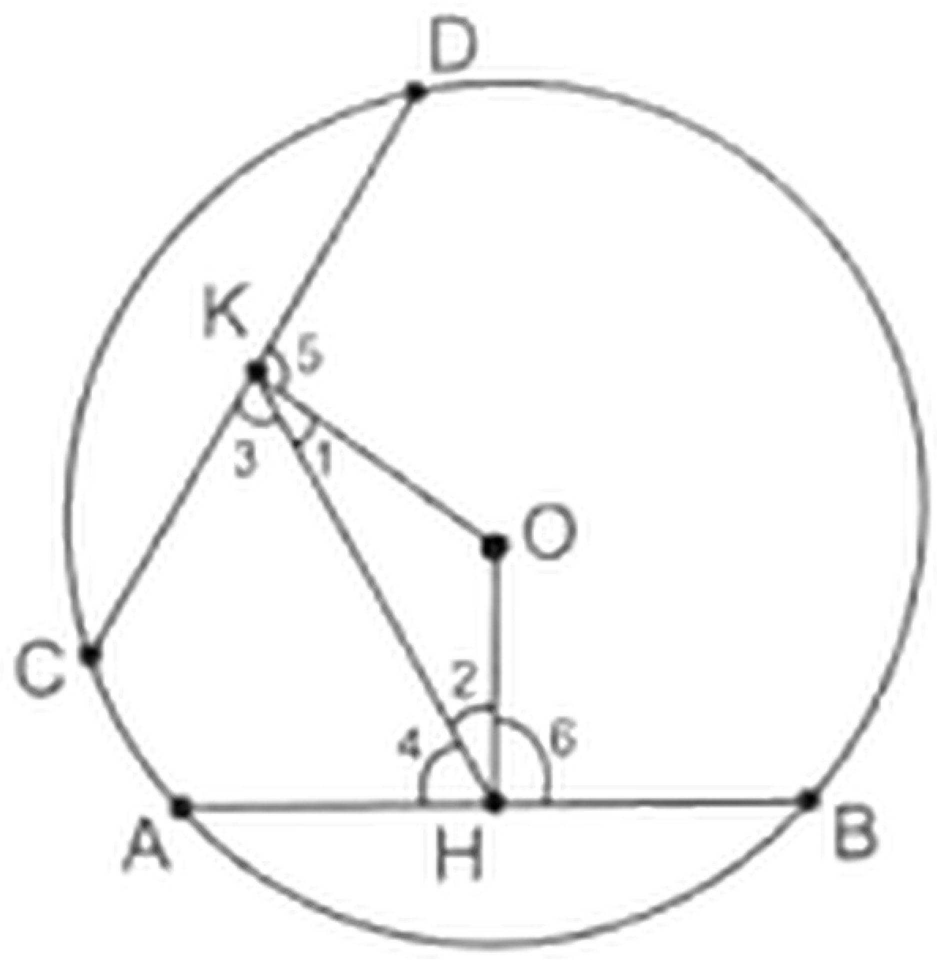
EXERCISE 10.2

I. AB and CD are two equal chords in a circle with center O. H and K are respectively the mid points of the chords. Prove that HK makes equal angles with AB and CD.

Solution:

Given:

$\overline{AB}$ and $\overline{CD}$ are equal chords of a circle with center O.

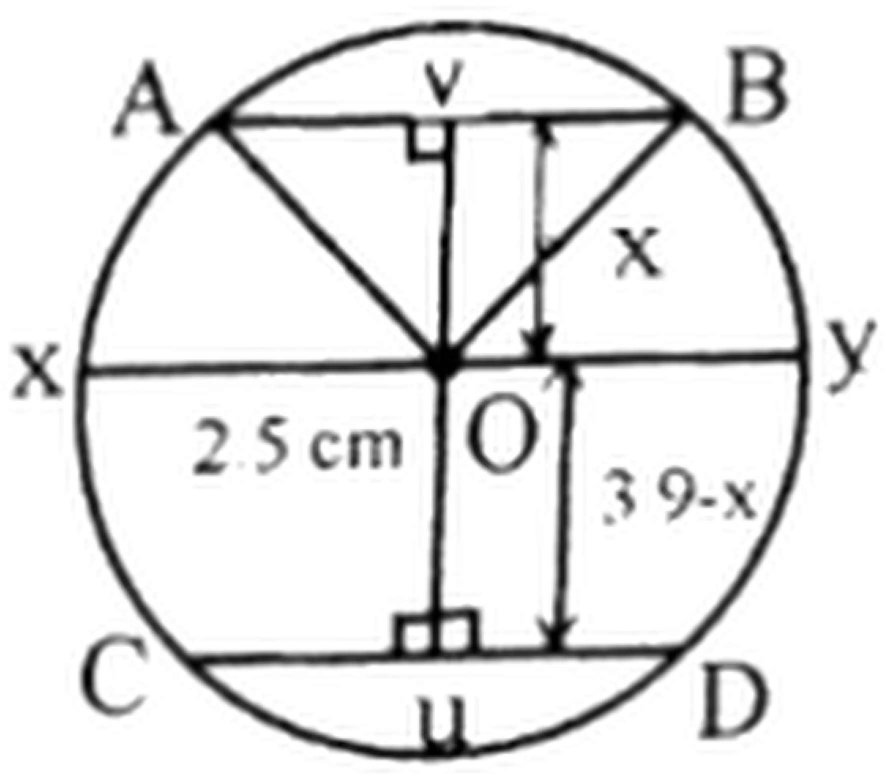


To prove:

(i) m∠AHK = m∠CKH

(ii) m∠BHK = m∠DKH

Statements	Reasons
In △HOK	
$m\overline{OH} = m\overline{OK}$	Radii of the circle.
∴ m∠1 = m∠2 _____(i)	
Also m∠5 = m∠6 _____(ii)	Each of 90°
m∠1 + m∠5 = m∠2 + m∠6	adding (i) and (ii)
m∠DKH = m∠BHK	Proved
m∠AHO = m∠CKO _____(iii)	Each of 90°
m∠2 = m∠1 _____(iv)	
m∠AHO - m∠2 = m∠CKO - m∠1	Subtract (iv) from (iii)
m∠AHK = m∠CKH	Proved



Solution:

Given:

$m\overline{OY} = m\overline{OX} = 2.5cm$

$m\overline{UV} = 3.9cm$

$m\overline{AB} = 1.4cm$

Required:

$m\overline{CD} = ?$

In triangle OAV

$$m\overline{OA}^2 = m\overline{OV}^2 + m\overline{VA}^2$$

$$2.5^2 = x^2 + (0.7)^2$$

$$\Rightarrow x^2 = 2.5^2 - 0.7^2$$

$$= 625 - 0.49 = 5.76$$

$$x = 2.4\ cm$$

$$m\overline{OU} = 3.9 - 2.4 = 1.5\ cm$$

In $\triangle OUC$

$$m\overline{OC}^2 = m\overline{OU}^2 + m\overline{CU}^2$$

$$2.5^2 = 1.5^2 + m\overline{CU}^2$$

$$\Rightarrow m\overline{CU}^2 = 2.5^2 - 1.5^2$$

$$= 6.25 - 2.25 = 4$$

$$\overline{CU}^2 = 4 \Rightarrow \overline{CU} = \sqrt{4} = 2$$

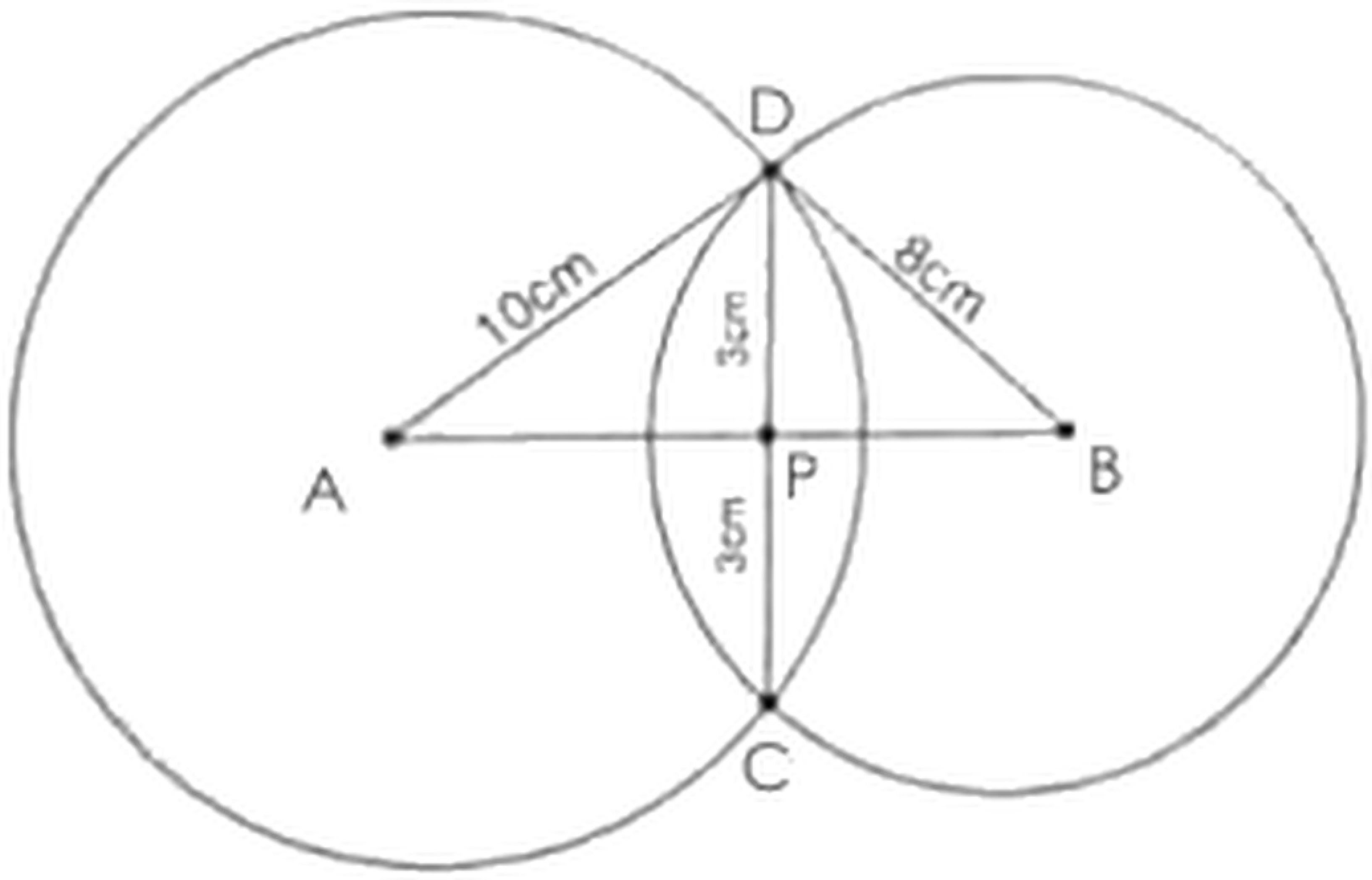
$$m\overline{CD} = m\overline{CU} + m\overline{UD}$$

$$m\overline{CD} = 2 + 2$$

3. The radii of two intersecting circles are 10cm and 8cm. If the length of their common chord is 6cm then find the distance between the centers.

Solution:

Given:



$m\overline{AD} = 10cm$, $m\overline{BD} = 8cm$,

$m\overline{DC} = 6cm$

Required:

$m\overline{AP} = ?$, $m\overline{PB} = ?$

In $\triangle$ ADP.

$m\overline{AD}^2 = m\overline{DP}^2 + m\overline{AP}^2$ $\because m\overline{AD} = m\overline{AP}$

$(10)^2 = (3)^2 + mAP^2$

$\Rightarrow mAP^2 = 100 - 9 = 91$

$\Rightarrow mAP^2 = \sqrt{91}\,cm = 9.54cm$ (approx)

and In $\triangle$DPB.

$m\overline{BD}^2 = m\overline{DP}^2 + m\overline{PB}^2$

$8^2 = 3^2 + m\overline{BP}^2$

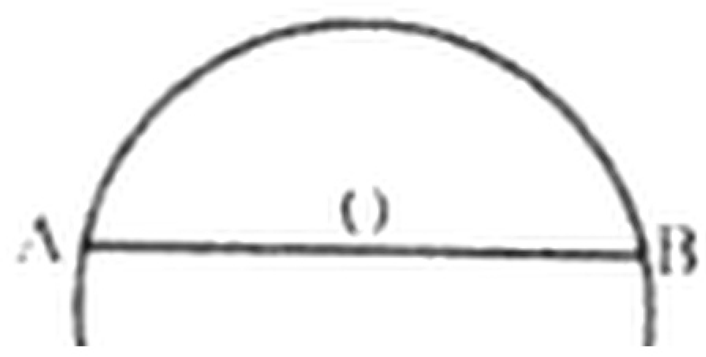
$\Rightarrow m\overline{BP}^2 = 64 - 9 = 55$

$m\overline{BP}^2 = \sqrt{55}\,cm = 7.42cm$(approx.)

So, the distance between the centers $= m\overline{AP} + m\overline{BP} = 9.54 + 7.42 = 16.96$ cm

4. Show that greatest chord in a circle is its diameter.

Solution:



To prove:

AB > CD

Or greater than any other chord.

Proof:

∴ AB is nearer the center the CD.

∴ AB > CD

Hence, AB, being nearest the center then all chords. So, AB is greater than any one of them.