Exercise 10.1

1. Prove that the tangents drawn at the ends of a diameter in a given circle must be parallel and conversely.

Solution:

Given:

Let ℓ and m be two tangents to the circle at the end points of a diameter $\overline{PQ}$

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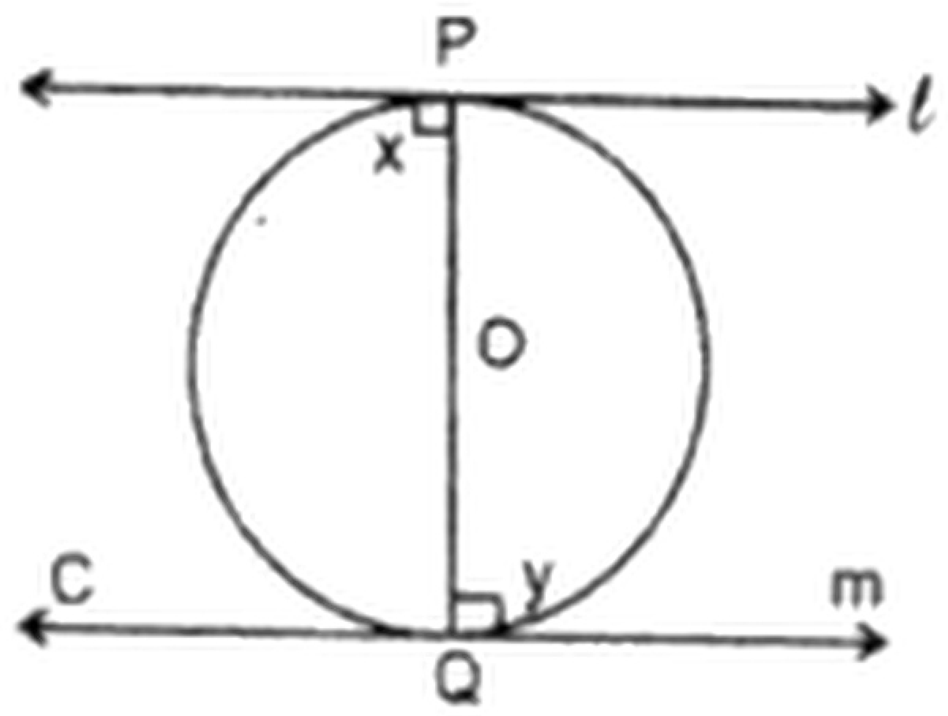


To prove: $\ell \parallel m$

Proof:

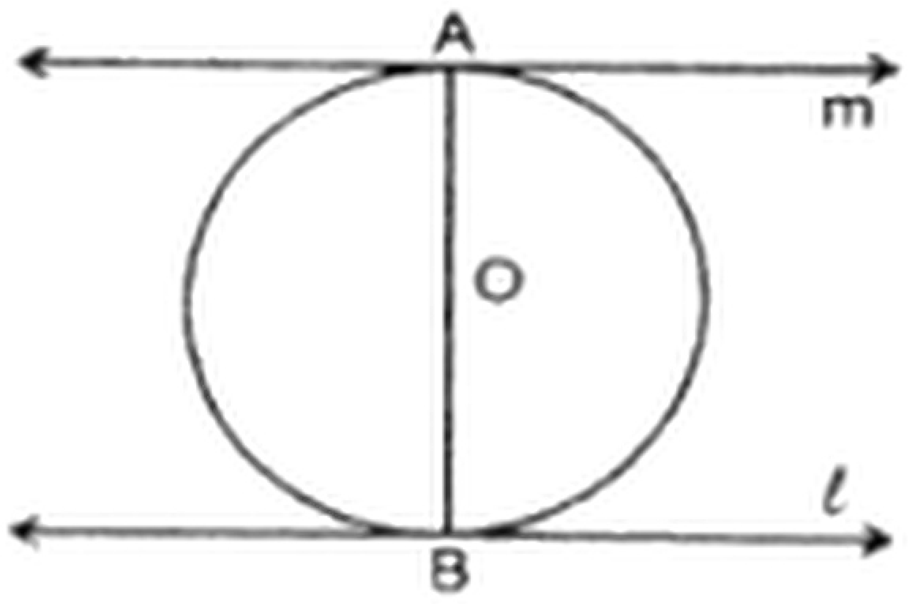
Statements	Reasons
$OP \perp \ell$, $OQ \perp m$	∴ A tangent at any point of a circle is ⊥ to the radius through the point of contact.
$\angle x = 90^\circ$, $\angle y = 90^\circ$	
$\Rightarrow m\angle x = m\angle y = 90^\circ$	
But:	
$m\angle x$ and $m\angle y$ are alternate angles.	
Hence, $\ell \parallel m$	

Conversely: parallel tangents of a circle must pass through its center.

Given:

Let ℓ and m are tangent to the circle at the ends of diameter $\overline{AB}$.

To the center O, and AB ⊥ ℓ and AB ⊥ m.



To prove:

$\overline{AB}$ passes through the center (diameter)

Proof:

If $\overline{AB}$ does not pass through the center join $\overline{OB}$.

$\overline{OB}$ is radius and ℓ is a tangent at B.

So that

$\overline{OB} \perp \ell$ or

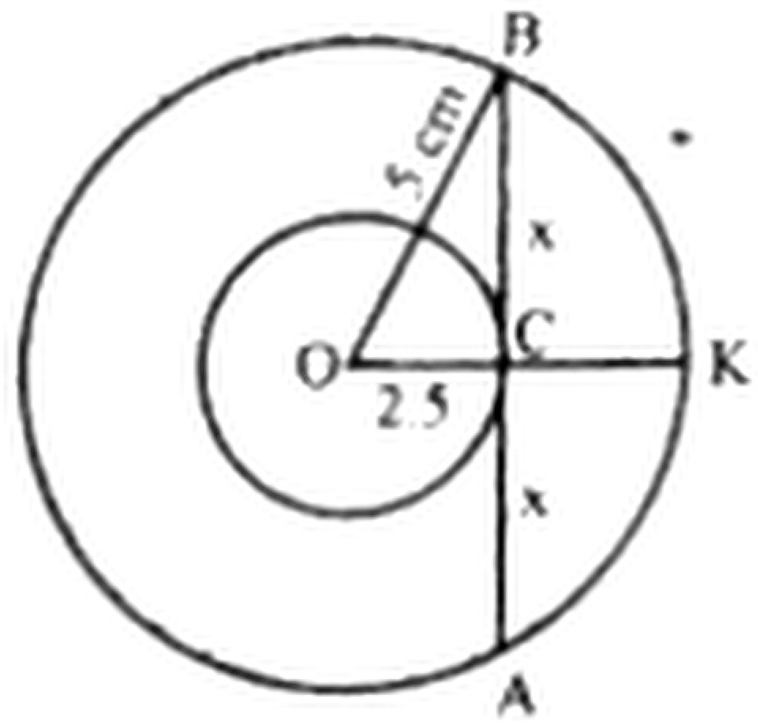
But $\overline{AB} \perp \ell$. (given)

$\therefore$ $\overline{OB}$ coincides with $\overline{AB}$.

Hence, $\overline{AB}$ passes through the center.

2. The diameters of two concentric circles are 10 cm and 5cm respectively. Look for the length of any chord of the outer circle which touches the inner one.

Solution:



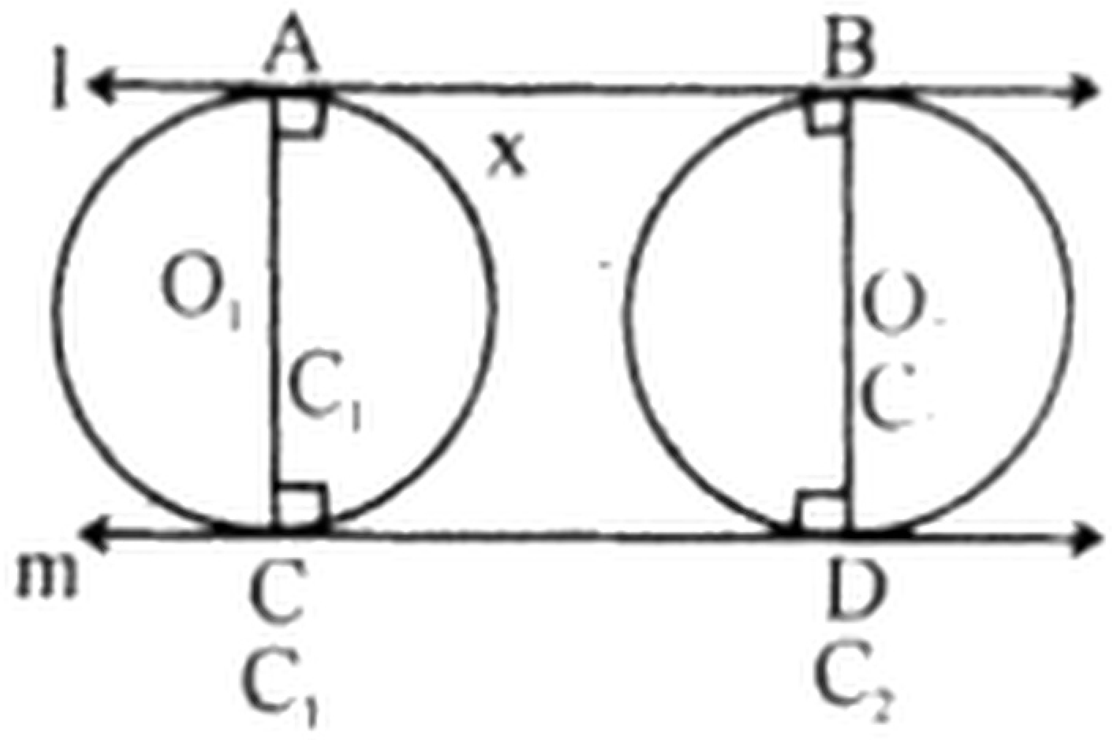
In a triangle OCB.

$$\left(\overline{OB}\right)^2 = \left(\overline{OC}\right)^2 + \left(\overline{CB}\right)^2$$

$$\Rightarrow \left(\overline{CB}\right)^2 = \left(\overline{OB}\right)^2 - \left(\overline{OC}\right)^2$$
$$= (5)^2 - (2.5)^2$$
$$= 25 - 6.25$$
$$= 18.75$$
$$\Rightarrow \quad \overline{CB} = \sqrt{18.75}$$
$$\overline{AB} = 2\overline{CB} = 2\sqrt{18.75} = 8.7\text{cm}$$

3. AB and CD are the common tangents drawn to the pair of circles. If A and C are the points of tangency of 1st circle where B and D are the points of tangency of 2nd circle, then prove that AC II BD.

Solution:



Given:

Two circle C_1 and C_2. Points of tangency of C_1 is A and C and points of tangency of C_2 is B and D

To Prove:

AC ∥ BC

Proof:

Statements	Reasons
In circle "C_1"	
And $\ell \parallel m$	Tangent is perpendicular to the circle
$m\angle CAB = 90°$ ______(i)	
and in circle "C_2"	
$\ell \parallel m$	
and $m\angle ABC = 90°$ ______(ii)	Proved
$\Rightarrow \angle CAB \cong \angle ABD$	Tangent is perpendicular to the circle
Similarly $\angle ACD \cong BDC$	by (i) & (ii)

Therefore: ABCD is rectangle $\therefore \overline{AC} \parallel \overline{BC}$	Parallel sides of a rectangle.